

We will discuss some of problems and modern tools of discrete probability. Topics will be some of the following (not necessarily in the order appearing below):

1. Random walks: their relation with electrical networks, Green's function, harmonic potential. Lyons and Peres, Probability on trees and networks, Ch 2 (some of the material is also covered in Levin-Peres, Chapters 9 and 10).
2. Analysis of Markov chains: Spectral theory, mixing times, cutoff phenomenon, isoperimetric estimates, comparison techniques, couplings. References: (1) Levin-Peres, mixing times of Markov chains. Potential additional topics: Characterization of cutoff, Random walks on nilpotent groups, random walks on random Cayley graphs.
3. Random graphs and percolation: Erdos-Renyi random graphs (in particular, the emergence of the giant component), random regular graphs, Galton-Watson trees. Possibly also preferential attachment, small world graphs and Random Cayley graphs.
4. Ergodic theory: Birkhoff's ergodic Theorem and Kingman's subadditive ergodic theorem. Applications.
5. Percolation: Russo-Seymour-Welsh technology, the Harris-Kesten theorem, critical and off-critical behaviour. Possibly conformal invariance (Smirnov's theorem).
6. Introduction to statistical mechanical models: The Ising and the Potts models.