THE UNIVERSITY OF BRITISH COLUMBIA

Greg Martin

Professor
Department of Mathematics

Lectures: Tuesdays and Thursdays, 11:00 AM–12:20 PM, room MATH 202 (Mathematics Building). *The first day of class will be Tuesday, September 10.*
Office hours: right after class, by appointment, or drop in
Office: room MATH 212 (Mathematics Building)
Email address: gerg@math.ubc.ca

Lecture notes:

- Tuesday, September 10 and Thursday, September 12
- Tuesday, September 17 and Thursday, September 19
- Tuesday, September 24 and Thursday, September 26
- Tuesday, October 1 and Thursday, October 3

Course description: The first few weeks will be spent quickly covering the foundations of elementary number theory: divisibility, congruences, prime numbers, and so on, some of which might already be familiar to you. Emphasis will be on a level of mastery sufficient for you to teach the material. Once we have this foundation, we will move on to roots of polynomial congruences, arithmetic and multiplicative functions, binary quadratic forms, and parametrizing Pythagorean triples. The two most important topics of the course are primitive roots and quadratic reciprocity. Topics that might also be covered include simple Diophantine equations, Diophantine approximation, and continued fractions.

Course textbook: *An Introduction to the Theory of Numbers*, by Niven, Zuckerman, and Montgomery, 5th edition (recommended). We will cover roughly the following sections:

- Sections 1.2–1.3: divisibility, gcds, primes
- Sections 2.1–2.3 and 2.6–2.8: congruences, solutions of congruences, Chinese remainder theorem, prime power modulus, prime modulus, primitive roots and power residues
- Sections 3.1–3.6: quadratic residues, quadratic reciprocity, Jacobi symbol, sums of two and four squares
  - (if time permits: binary quadratic forms)
- Sections 4.2–4.4: arithmetic functions, Möbius inversion, recurrence functions
  - (if time permits: A.4, linear recurrences)
- Sections 5.1 and 5.3: solving $ax + by = c$ in integers, Pythagorean triangles
  - (if time permits: 5.2 and 5.4, simultaneous linear equations and assorted examples)
- Sections 6.1–6.3: Farey sequences, rational approximations, irrational numbers (as time permits)
- Sections 7.1–7.8: the Euclidean algorithm, uniqueness, infinite continued fractions, (best possible) approximations of irrationals by rationals, periodic continued fractions, Pell's equation (as time permits)

Notes to undergraduates: For all practical purposes, *MATH 437 is an honours course*! It treats roughly the same material as MATH 312 and 313 combined, and will take nearly twice as much work as either of those classes. Note that a student cannot have credit for both MATH 312 and MATH 437, nor for both MATH 313 and MATH 437. To enroll in MATH 437, an undergraduate student must have already taken, or be taking simultaneously, one of MATH 320 or MATH 322.

The word “elementary” in the title does not mean the course isn't difficult; rather it means that the course doesn't use techniques from real or complex analysis or from abstract algebra. The course will not require any particular background in number theory. What is required is “mathematical sophistication”, which certainly includes being able to understand and write proofs. Be forewarned that this course will be taught at the level of a graduate course. Honours students typically will be well-equipped to succeed in this course.

Evaluation: The course mark will be based on (approximately) weekly homework assignments (80% of the final mark; the lowest homework score will automatically be ignored) and engaged participation in in-class group problem sessions (20% of the final mark).

Homework: Your homework will be marked on correctness, completeness, rigor, and elegance. A correct answer will not earn full marks unless it is completely justified, in a rigorous manner, and written in a logical sequence that is easy to follow and confirm. I plan on being pedantic about completeness of solutions (for example, if you invoke Euler's theorem to assert that $a^{\varphi(q)} \equiv 1 \pmod{q}$, you need to explicity acknowledge the fact that a must be relatively prime to q). Part of the goal of this course is to provide training and practice at writing proofs with sufficient rigor to be accepted by research journals.

You are very welcome to come by my office (or email me) and ask questions about the lecture material, homework/group work problems, clarity of style in proof writing, or related mathematical content. You may make an appointment with me via email; you are also welcome to just drop by my office and see if I'm there—about half the time, I'll be free to talk right then. Students are allowed to consult one another (or other sources) concerning the homework problems, but *your submitted solutions must be written by you in your own words*. If two students submit virtually identical answers to a question, both can be found guilty of plagiarism.

All homework assignments and any other course materials will be posted on this course web page below. ***Homework solutions must be prepared in LaTeX*** and submitted to me in PDF format via email; please add your name to the filename before submitting your homework (for example, GregMartin-homework1.pdf). I will supply LaTeX templates with each assignment. All homeworks are due before the beginning of Tuesday's class (10:59 AM).

Group work: Group work will take place from approximately 11:30 AM–12:20 PM every Tuesday in class. I will have rotating group assignments prepared so that you can work with everyone over the course of the semester; I will hand out the group problems once people are in their groups. Group work is essentially graded on attendance and participation; you will not have to write up any solutions or summary of your discussions. I will post solutions to the group work below after class every Tuesday. I will assume that everyone taking/auditing the course will be present for every group work; please ***let me know in advance if you will ever miss a group work day***, so that I can prepare the groups accordingly (and often will be willing to excuse your absence for grading purposes).

- Homework #0: due Thursday, September 19. Download both the TeX file and the PDF file.
- Homework #1: due Tuesday, September 24
- Homework #2: due Tuesday, October 1
- Homework #3: due Tuesday, October 8
- Homework #4: due Tuesday, October 15
- Homework #5: due Tuesday, October 22
- Homework #6: due Tuesday, October 29
- Homework #7: due Tuesday, November 5
- Homework #8: due Tuesday, November 19
- Homework #9: due Tuesday, November 26
- Homework #10: due Tuesday, December 3

- Group Work #1: Tuesday, September 17
- Group Work #2: Tuesday, September 24
- Group Work #3: Tuesday, October 1
- Group Work #4: Tuesday, October 8
- Group Work #5: Tuesday, October 15
- Group Work #6: Tuesday, October 22
- Group Work #7: Tuesday, October 29
- Group Work #8: Tuesday, November 5
- (no class on Tuesday, November 12)
- Group Work #9: Tuesday, November 19
- Group Work #10: Tuesday, November 26

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