Math 529: Differential Topology

Course Outline

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Course description

This graduate course continues the study of smooth manifolds, with differential forms providing a common language for topology and geometry. We begin with de Rham cohomology and Poincaré duality, then develop vector bundles, connections, curvature, and the Chern–Weil construction of characteristic classes. We next study differential-form models of equivariant cohomology and localization. Time permitting, the course concludes with an introduction to Yang–Mills theory.

Background and course format

Math 525 is the recommended preparation, but some familiarity with manifolds is sufficient. For example, students who have encountered manifolds in an algebraic topology or algebraic geometry course should have enough background. No prior knowledge of differential forms is assumed. The course is intended to be paced somewhat faster than a first-year graduate course, while remaining more introductory than a graduate topics course.

For the first few weeks, lectures will follow Bott and Tu's *Differential Forms in Algebraic Topology*. Subsequent lectures will primarily follow the instructor's notes. The books and articles on the next page provide supporting explanations, examples, and further reading.

Tentative sequence of topics

1. **Differential forms and de Rham cohomology.** The de Rham complex, homotopy invariance, compactly supported cohomology, and the Mayer–Vietoris sequence.
2. **Poincaré duality.** Integration and the cohomological pairing; the relation between differential forms, orientations, and intersection theory.
3. **Bundles, connections, and curvature.** Vector bundles and principal bundles; connections, covariant derivatives, curvature, and the Bianchi identity.
4. **Chern–Weil theory.** Construction of characteristic classes from invariant polynomials in curvature; independence of the connection and examples of characteristic forms.
5. **Equivariant cohomology and localization.** The homotopy quotient and its differential-form models; the Weil and Cartan models, equivariant characteristic classes, and the Atiyah–Bott–Berline–Vergne localization formula.
6. **Yang–Mills theory (time permitting).** Gauge transformations, the Yang–Mills functional and equations, and selected examples or applications.

Recommended reading

The principal references are listed first. Readings will be selected to complement the lectures; students are not expected to purchase all of these books.

Principal references

Raoul Bott and Loring W. Tu *Differential Forms in Algebraic Topology*. Graduate Texts in Mathematics 82, Springer, 1982.

The reference for the opening weeks: de Rham theory, Poincaré duality, and the topological interpretation of differential forms.

Loring W. Tu *Differential Geometry: Connections, Curvature, and Characteristic Classes*. Graduate Texts in Mathematics 275, Springer, 2017.

A supporting text for bundles, connections, and Chern–Weil theory. Chapters 5–6, on vector bundles and principal bundles with characteristic classes, are particularly relevant.

Loring W. Tu *Introductory Lectures on Equivariant Cohomology*. Annals of Mathematics Studies 204, Princeton University Press, 2020.

The main supporting text for equivariant cohomology: Part II treats the geometry of principal bundles; Part III develops the Weil and Cartan models; Parts IV–V treat localization and applications.

Supplementary books

Victor W. Guillemin and Shlomo Sternberg *Supersymmetry and Equivariant de Rham Theory*. Springer, 1999.

A complementary treatment of the Weil and Cartan models, equivariant characteristic classes, Thom classes, and localization.

John W. Milnor and James D. Stasheff *Characteristic Classes*. Annals of Mathematics Studies 76, Princeton University Press, 1974.

A topological reference for vector bundles and characteristic classes. Appendix C, "Connections, Curvature, and Characteristic Classes," provides a compact account of the differential-geometric construction.

Gregory L. Naber *Topology, Geometry and Gauge Fields: Foundations*. Second edition, Texts in Applied Mathematics 25, Springer, 2011.

Optional reading for Yang–Mills theory. The chapter "Gauge Fields and Instantons" develops connections, curvature, the Yang–Mills functional, and instantons through examples.

Short readings available online

Eckhard Meinrenken, "Equivariant cohomology and the Cartan model." A nine-page overview connecting the Borel construction, differential-form models, and localization.

Raoul Bott and Loring W. Tu, "Equivariant characteristic classes in the Cartan model," arXiv: math/0102001 (2001). A bridge from Chern–Weil theory to equivariant characteristic classes, comparing the differential-geometric and topological constructions.