Acknowledgement

UBC's Point Grey Campus is located on the traditional, ancestral, and unceded territory of the xʷməθkʷəy̓əm (Musqueam) people. The land it is situated on has always been a place of learning for the Musqueam people, who for millennia have passed on knowledge of their culture, history, and traditions from one generation to the next on this site.

Course Information

Galois theory is named after Évariste Galois (1811-1832), a French mathematician, who did not live to be 21! His collected works, running to around two dozen pages, revolutionized modern mathematics through his fundamental discoveries and insights.
URL: https://canvas.ubc.ca/courses/191281
TA: Shadgar, Negin ; email: negin@math.ubc.ca

Course Title	Course Code Number	Credit Value
Field Theory and Galois Theory	MATH 422/501	3

Time and Location:

Lecture	Time	Location
Tue , Thurs	9:30 -11	ESB 4122 (Earth Sciences Building)

Prerequisites

MATH 322, MATH 323

Contacts

Course Instructor(s)	Contact Details	Office Location	Office Hours
Sujatha Ramdorai	• Email: sujatha@math.ubc.ca • www.math.ubc.ca/~sujatha	Math Annex 1201	MW 3:30-5 PM by appointment

Course Structure

The course will be lecture-based.

Schedule of Topics

Lectures will naturally diverge somewhat from any schedule, so the schedule below may change slightly. Broadly, the course will consist of three parts:

Field Theory: 9 lectures
Topics: Field extensions, Finite and Algebraic Extensions, Minimal polynomials, Splitting fields and Normal Extensions, Automorphisms and Separable extensions, Finite fields, Primitive elements, Simple extensions, Purely inseparable extensions, Galois extensions.

Galois Theory: 9 Lectures
Topics: Fundamental Theorem of Galois theory, Galois group, Cyclic Extensions, Algebraic closures, Norm and Trace, Solvable and radical extensions, Nonabelian Kummer extensions, Discriminants and Resolvents, Polynomials and Solutions, Algebraic independence, Normal Basis Theorem, Roots of Unity, Hilbert Theorem 90 and Group Cohomology, Cyclotomic Extensions.

Applications: 7 Lectures
Transcendence of π and e, Ruler and Compass constructions, Solvability of Radicals, Infinite Extensions.

Learning Outcomes

At a very basic level, Galois theory is seemingly about finding solutions to equations. Galois brilliantly recognised that this is related to what today in modern algebra is called 'Group Theory'. Understanding this connection forms the fundamental theme of Galois theory. Along the way, we will also learn about the notion of transcendental and algebraic numbers, and the impossibility of doubling a cube or trisecting an angle. Apart from possessing an intrinsic beauty of its own, Galois theory today is fundamental to different areas within higher mathematics, such as Algebraic Geometry, Algebraic Number Theory, Arithmetic Geometry.

This is a Graduate Course in Mathematics and as such, proofs will form an important part of teaching and learning.

Learning Activities

Regular reading and working through proofs from the lectures, and solving problems in the assignments are expected from the students. Students are expected to participate by asking questions in class when they do not understand the material. Students are expected to fully understand the solutions to the exercises that they provide. The homework problems will be posted on the course webpage on Wednesdays, and are expected to be handed in by Thursday of the next week (10 days later).

Registered students can download the assignments from the course page in Canvas. Students should turn in the homework in typseset form, if at all possible. If they are handwritten, it is the student's responsibility to ensure clarity and legibility. Written work should be presented carefully, should be complete with sufficient details and requisite references to the material used in proofs. Students may be assigned some extra reading material for self-study, in which case the students are expected to demonstrate that they have learnt the material. This could be done by way of a short essay or presentation.

Students are permitted to use artificial intelligence tools, including generative AI, to gather information, review concepts or to help produce assignments. However, students are ultimately accountable for the work they submit, and any content generated or supported by an artificial intelligence tool must be cited appropriately. Use of AI tools is not permitted during midterm exams and final exams in this course.

Learning Materials

All information and material relevant to the course will be available on the Course page on Canvas. Notes of the lectures will be made available to the registered students, to the extent possible. There will be no single textbook used for the course. The topics are listed in the schedule. Here is a list of possible references:

- J.S. Milne, **Fields and Galois theory**, online lecture notes avialable at https://www.jmilne.org/math/CourseNotes/FT.pdf
- Ian Stewart, **Galois Theory**, (available on Amazon)
- Dummit and Foote, **Introduction to Abstract Algebra**.

Assessments of Learning

There will be one midterm exam, one final exam and up to six assignments. The course evaluation will be as follows.

- Final Exam (in class): 40 points
- Midterm Exam (in class) TBD: 30 points
- Assignments: 30 points

There will be no make-up midterm, quizzes or assignments in this course. Missing the midterm for a valid reason normally results in their weight being transferred to the final exam. Students may ask for up to two extensions for assignments which will be considered at the discretion of the instructor. Without permission, late homework will not be accepted (a mark of zero being assigned). Examples of valid reasons include illness and travel to play a scheduled game for a varsity team. The UBC Vancouver Senate's Academic Concession Policy V-135 applies to all assignments in this course, and students are advised to read this policy carefully.

Examples of reasons that are not valid include conflicts with personal travel schedules or conflicts with work schedules. Any student who misses the midterm is to present a self-declaration form for reporting a missed assessment to the instructor within 72 hours of the midterm date. This policy conforms with the UBC Vancouver Senate's Academic Concession policy V135. Non-attendance at an exam/quiz (without a valid reason) will result in a mark of zero being assigned.

University Policies

UBC provides resources to support student learning and to maintain healthy lifestyles but recognizes that sometimes crises arise and so there are additional resources to access including those for survivors of sexual violence. UBC values respect for the person and ideas of all members of the academic community. Harassment and discrimination are not tolerated nor is suppression of academic freedom. UBC provides appropriate accommodation for students with disabilities and for religious observances. UBC values academic honesty and students are expected to acknowledge the ideas generated by others and to uphold the highest academic standards in all of their actions. Details of the policies and how to access support are available on the `https://senate.ubc.ca/policies-resources-support-student-success`, which is the UBC Senate website.

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