MATH_V 317 951 2026SS Calculus IV

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DESCRIPTION:

In this course, we study the calculus of vector-valued functions of several variables. We will study parametrization, differentiation and integration, length and area on curves and surfaces. We will study vector fields and their operations grad, div, and curl. We will study integral theorems of Green, Gauss, and Stokes for vector fields.

TOPICS:

- Differential geometry of Curves
- Vector fields
- Line integrals
- Green's theorem
- Parametric Surfaces
- Surface integrals
- Gradient, divergence, and curl
- Divergence theorem
- Stokes's theorem
- Classical equations from physics

Prerequisites: One of MATH 200, MATH 226, MATH 253. MATH 221 is recommended.
Textbook: CLP-4 Vector Calculus by Feldman, Rechnitzer and Yeager. Available here (https://www.math.ubc.ca/~CLP/CLP4/) .

Lecturer: Soheil Akbari
Lecture time: MW 10:00AM-12:00N and TTh 10:00-11:30AM. In Mathematics Annex (MATX) | Floor: 1 | Room: 1100.
Office hours: ...
Grade breakdown

20% Homework
30% Midterm
50% Final exam

IN TERM ASSESSMENTS:

The homework schedules are indicated in the bi-weekly schedule. The Homework sets will be posted directly on this page. These must be typed or scanned and submitted on the Canvas page. You may work in groups. However, you must write up your own solutions.

WEEKLY SCHEDULE

Week1 (Jul 6-9);

This week corresponds roughly to Section 1.1-1.4 in CLP-4. NOTE: Their discussion is restricted to plane curves while we discuss space curves as well, viewing plane curves as a special case. They also introduce curvature slightly differently than we do. They introduce the arc length parameter here while we defer this to week 2.

Some complementary notes for week 1:
1- The orientation of the curve is given by the direction of the tangent vector T(t): $T(t)=dr/dt$
When reparameterizing the curve, the new tangent vector with respect to the parameter u is:
T(u)=$dr/du=dr/dt \cdot dt/du$
To maintain the same orientation, the direction of the tangent vector should not change when we switch from parameter t to parameter u. This means:

$T(u)=T(t) \cdot dt/du$

For $T(u)$ to have the same direction as $T(t)$, the scalar dt/du must be positive. If $dt/du>0$, then $T(u)$ points in the same direction as $T(t)$. If dt/du<0, then $T(u)$ would point in the opposite direction to $T(t)$, thus reversing the orientation of the curve.

Why Assume $dt/du>0$?
By assuming $dt/du>0$, we ensure that the reparameterization does not change the direction of the tangent vector and hence the orientation of the curve. This assumption is crucial for maintaining consistent geometric properties and ensuring that the curve's orientation is preserved under different parameterizations.

2- Translation of vectors usually refers to shifting all points of a vector by the same amount in space, which does not affect the direction vectors (tangent and normal). Thus, translation does not alter the derivatives r′(t) and $r''(t)$, and hence does not change the curvature and torsion.

3- If the curvature κ=0, the curve is a straight line where no bending occurs. In such cases, the torsion τ is not defined. Since the curvature is zero, the concept of torsion does not apply because there is no "plane of curvature" to twist out of. Hence, the torsion cannot be calculated for curves with zero curvature.

Week2 (Jul 13-16)

This week corresponds to sections 2.1, 2.3 and 2.4 in CLP-4.

- Curves, Frenet-Serret formulae and fundamental Theorem of curves, LINE INTEGRALS (definitions)

- Line integrals, interpretations & examples

- Line integrals, Conservative Vector Fields F

- F conservative => "curl F =0"

- "curl F =0" in U <=>$\int_C \boldsymbol{F} \cdot \boldsymbol{dr}$ path independent in U <=> F conservative in U): *each of these implications has some assumption on the set U.

Week3 (Jul 20-23)

For a simply connected region R (i.e. a region with no holes), the following can be shown (The following statements are equivalent for a simply connected region R in R^2) :
(a) f(x, y) = P(x, y) i+Q(x, y) j has a smooth potential Phi(x, y) in R
(b) Line integral of f · dr over curve C is independent of the path for any curve C in R.
(c) Line integral of f · dr over curve C = 0 for every simple closed curve C in R.

(d) $\partial P/\partial y = \partial Q/\partial x$ in R (in this case, the differential form P dx+Q dy is exact, i.e. there exists a function $f(x,y)$ such that: $df = \partial f/\partial x \ dx + \partial f/\partial y \ dy = P \ dx + Q \ dy$)

- Green's Theorem:

We should know

I. Intuitive understanding of line integrals.
II. Conservative fields: definition, finding a potential, their line integrals.
III. Equivalence of following three when domain of F is simply connected (which you may assume to be the case for quiz): **a)** curl **F**=0; **b)** path independence of $\int_C \mathbf{F} \cdot d\boldsymbol{r}$ (equivalently, integral is zero when C is closed loop); **c) F** is conservative,

(for 2D fields, replace a) with (F_1)_y-(F_2)_x =0)

The following problems from CLP-4 2.4 provide practice in these topics:

Q2 (conservative field or not?)
Q3 (line integral of a conservative field)
Q4 (simple addition of paths")
Q5 (F conservaitve => line integral around closed loops is zero (no condition on domain of F here)

Week4 (Jul 27-30) MIDTERM (Monday July 27: regular class time and place)

Our treatment of surfaces this week comes from Section 3.1-3.3 of the textbook CLP4 (in equations 3.3.1-3.3.3, we only treated formulae for dS appearing there, but not yet for $\hat{\mathbf{n}}dS$).

- 2D divergence Theorem, Parametric surfaces

- Parametric surfaces: coordinate curves, surface tangent plane and normal vector)

- Curvature of a surface: brief introduction to curvature of a surface and is not treated in the textbook, and this lecture is meant as a supplement and will not appear on HW or final exams.

- Integration of scalar functions on surfaces

- Integration of scalar functions on surfaces

Midterm

When: Monday-July 27, regular class time (90 minutes from 10:00 to 11:30)

Allowances: You may bring one sheet of regular size paper with anything you like written on either side of it.

Topics:

Curves:

-parametrizations (re-parametrizations, chain rule)

-curvature, torsion

-arc length parameter and parametrization

-the Frenet-Serret Frame {T, N, B} along a curve (you do not need to know the Frenet-Serret formulae).

Line integrals:

-Line integral of a vector field along a curve (will not be tested on line integral of a scalar function)

-Conservative fields: definition, finding a potential, their line integrals

-Know that when domain of F is open and simply connected, following are equivalent:

1) Curl F =0 (replace with 2D curl F=0 for 2D fields).

2) $\int_C \mathbf{F} \cdot dr$ is path independent (or equivalently, =0 when C closed)

3) F is conservative

Practice: Review our notes, the examples within, our HW's and the relevant parts of the text.

Week5 (Aug 3-6)

- Integration of vector fields across surfaces (Flux Integrals)

- Flux integrals (examples); general formulae for graphs

- General formulae for level surfaces

- Divergence Theorem

- Divergence Theorem

- Proof of Divergence Theorem

Week6 (Aug 10-13)

- Interpretations of divergence, Stokes Theorem

- Stokes Theorem

- Currently missing, if you need this right away then please find a classmate who took notes for this lecture

-Proof of Stokes Theorem, interpretations of curl F

- "div F =0" in U <=> F has a vector potential in U (ie, F is the curl of some vector field G) <=> $\int\int_S \mathbf{F} \cdot \hat{n} dS$ is surface inpendent" in U
*note: first implication assumes U is convex.

- Heat equation, Navier-Stokes equations)

- Navier-Stokes equations)

- Maxwell equation in electrostatics)

COURSE POLICIES:

• Missing/late homework: No late homework will be accepted. You can receive one concession during the term by submitting a Department of Mathematics self-declaration form (which can be found **here (https://owncloud.math.ubc.ca/index.php/s/mumsWsIjdjR1idJ)**). The weight of the missed homework with accepted concession form will be transferred to the other assignments. More information on UBC's policy for academic concessions can be found here.

• Missing midterms: There are no make-up tests in this course. A student who misses a midterm for a valid reason must present to their instructor a Department of Mathematics self-declaration form, and the weight of the assessment will be transferred to the final exam. More information on UBC's Academic Concession Policy can be found here.

• Missing the Final Exam: You will need to present your situation to the Dean's Office of your Faculty to be considered for a deferred exam.

FINAL EXAM INFORMATION

Day: ... August, 2026
Time: ...
Location: ....

Bring photo identification as we will verify your ID.

You may bring a double-sided 8.5 x 11 cheatsheet into the exam. You should be able to read your cheatsheet with use of your regular eyewear if any (no magnifying glass..etc). Calculators and cell phones are not allowed.