THE UNIVERSITY OF BRITISH COLUMBIA
Sessional Examinations   April 2005
Mathematics 414

Time: $2\frac{1}{2}$ hours

Please do 5 questions. Solutions should be very full, and aimed at as low a grade level as possible. Calculators are not to be used. Answers can be left in "calculator-ready" form.

1. (a) In how many ways can 101 identical muffins be distributed between A, B, and C? Do not assume that everyone gets at least one muffin.

(b) What about between A, B, C, and D?

2. The vertices of a square, taken counterclockwise, are $A(11,0)$, $B(s,t)$, $C(0,\pi)$, and D. Find (s,t).

3. Find efficiently the product of all the positive integers that divide 1600.

4. Given that a is positive and $a^2 + \dfrac{1}{a^2} = 3$, find $a^3 + \dfrac{1}{a^3}$ and $a^3 - \dfrac{1}{a^3}$ efficiently. Give as simple answers as possible.

5. Describe in detail how to cut up an equilateral triangle into (a) 7 equilateral triangles; (b) 8 equilateral triangles; (c) 1001 equilateral triangles.

6. Sketch, with detailed justification, the part of the xy plane that satisfies the inequality $|y - x| + |y - 2x| \le 6$, and find its area.

Note that for any positive k, the region $|y - x| + |y - 2x| = 6k$ is a scaled version of the region $|y - x| + |y - 2x| = 6$. (I should have pointed out this way of dealing with the inequalities in the solutions of the similar workshop problems.)

7. How many sequences a_0, a_1, a_2, a_3, a_4, a_5 of six non-negative integers are there such that (i) $a_{i+2} = 2a_{i+1} + a_i$ for all $i \le 3$ and (ii) $a_5 = 1000$?