Final Exam
Math 342 Section 101 December 11th 2010

Name ______________________________**Student Number** ______________________________

Signature ____

The exam is 150 minutes long and worth a total of 100 points. No books, notes or calculators may be used. **Show all of your work and justify your answers carefully**. You will be graded on the clarity of your exposition as well as the correctness of your answers.

Good luck.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total	

1 (10 points)**.** *Inverse elements:* Calculate the inverse of the following elements in $\mathbb{Z}/18\mathbb{Z}$.

(a) $[11]_{18}$

(b) $[7]_{18}$

(c) $[15]_{18}$

2 (10 points)**.** *Group theory:* The following set of 2×2 matrices

$$m_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \; m_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \; m_3 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \; m_4 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$m_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; m_6 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \; m_7 = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \; m_8 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

with matrix multiplication forms a group, G.

(a) (2 points) List the elements of its subgroup, K, generated by the element $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

(b) (8 points) Compute the *left* cosets of K.

3 (15 points)**.** *Error detecting code ISBN-13:* Consider the 10-ary code of length 13, using the alphabet $\mathbb{Z}/10\mathbb{Z}$ and following encoding algorithm.

ENCODE: $x_1 x_2 \cdots x_{12} \mapsto x_1 x_2 \cdots x_{12} x_{13}$ by

$$\sum_{i\ odd} x_i + \sum_{i\ even} 3x_i \equiv 0 (\mod 10).$$

Answer the following questions.

(a) (7 points) Find the missing digit of the ISBN-13 that is 97803 ? 6406156.

(b) (8 points) Explain why this code can detect one error.

4 (15 points)**.** *Coding theory:*

(a) (6 points) Show that the following two 4-ary *non-linear* codes are equivalent

$$C = \{0122, 1033, 2303, 3210\} \quad C' = \{0220, 1331, 2001, 3113\}.$$

(b) (9 points) Show that a q-ary $((q+1), M, 3)$ code satisfies $M \leq q^{q-1}$.

5 (15 points)**.** *Linear codes:* Consider the 3-ary linear code, C, with *parity check* matrix

$$\begin{pmatrix} 1 & 2 & 1 \end{pmatrix}.$$

(a) (10 points) List all the codewords in C. Determine how many errors C can detect.

(b) (5 points) Let D be the binary linear code with *parity check* matrix

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

Compute the coset leaders of D.

6 (15 points). *Order of an element:*

(a) (5 points) Define the *order* of a unit in a ring.

(b) (10 points) Given the ring $\mathbb{Z}/m\mathbb{Z}$, prove that

$$\textit{if } e \text{ is the order of a unit } [a]_m \in \mathbb{Z}/m\mathbb{Z} \text{ and } [a]_m^f = [1]_m \textit{ then } e \mid f.$$