

- (c) Argue that if n is very very large compared to m , say $n = m^4$, and A_N is sparse (containing 99% zeros in its entries), then the revised simplex method is much faster than the standard simplex method, even if we explicitly compute A_B^{-1} on each iteration.

[10] 2. Consider the problem (from the note sheet) maximize x_1 subject to $x_1 \leq 2$, $x_1 \leq \beta$, and $x_1 \geq 0$.

(a) Solve this LP for $\beta = 1$.

(b) For this final dictionary, write out \vec{x}_B , \vec{x}_N , and A_B . Find A_B^{-1} , and using A_B^{-1} write down a dictionary corresponding to the final dictionary where β is general (rather than $\beta = 1$).

(c) For what values of β is the dictionary in part (b) final?

(d) By performing one dual pivot, give a final dictionary for β slightly bigger than two 2.

(e) Try to do the same for β slightly less than 0. What happens?

- [10] **3.** For every real α find the value and equilibria strategies for the matrix game:

$$\begin{bmatrix} 1 & 2 \\ 3 & \alpha \end{bmatrix}.$$

Graph the value of the game as a function of α . Find the duality gap for the announce (pure strategy) game as a function of α .

- [10] 4. Consider the problem: maximize x_1 subject to $x_1 \leq 5$, $x_1 \geq 2$, $x_1 \geq 0$. Write this as a linear program in standard form. Use the two-phase method, **adding an auxiliary variable x_0 to EVERY slack variable equation in the dictionary**, to solve this LP. Use the smallest subscript rule to break any ties for entering or leaving variables.

- [10] 5. Consider our usual LP: maximize $4x_1 + 5x_2$ subject to $x_1 + 2x_2 \leq 8$, $x_1 + x_2 \leq 5$, $2x_1 + x_2 \leq 8$, and $x_1, x_2 \geq 0$. Write the slack variables for this linear program, and write down the dual linear program and dual slack variables.

Check to see if the following are optimal solutions to the primal linear program using complementary slackness:

(a) $x_1 = 3, x_2 = 2$;

(b) $x_1 = 2, x_2 = 3$;

- [10] **6.** Explain, using a formula on the note sheet involving A_B^{-1} , why the perturbation method never has a degenerate pivot. Explain exactly under what conditions you could get a degenerate pivot if you took one ϵ (i.e., $\epsilon = \epsilon_1 = \epsilon_2 = \dots$) instead of m different ϵ 's.

- [10] 7. You are given an $m \times n$ matrix game, A ,
- (a) Someone thinks that Alice has equilibrium strategy \vec{x} , and Betty \vec{y} . How could you verify this most quickly? Roughly how much time would this take?
 - (b) Someone thinks that Alice has equilibrium strategy \vec{x} , but has no idea what Betty's equilibrium. How could you verify this most quickly? Roughly how much time would this take?
 - (c) No one has any idea what the equilibrium strategies are. Outline the equilibrium solving approach in "Poker, Taxes, etc." that does not involve linear programming, and outline the linear programming approach.
 - (d) Explain how the linear programming approach, combined with guessing which variables (decision and slack) are non-essential, yields the first approach of part (c). ■

The End

Be sure that this examination has 14 pages including this cover

The University of British Columbia

Final Examinations - April 2009

Mathematics 340–202

Closed book examination

Time: 150 minutes

Name _____ Signature _____

Student Number _____ Instructor's Name _____

Section Number _____

Special Instructions:

THIS EXAM IS TWO-SIDED! You will be given note sheets. Calculators, other notes, or other aids may not be used. Answer questions on the exam.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

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(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

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3. Smoking is not permitted during examinations.

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