THE UNIVERSITY OF BRITISH COLUMBIA

Sessional Examinations—April 2007

Mathematics 335

Time: $2\frac{1}{2}$ hours

This examination has 2 pages. Please do *four* of the six parts of question A, and *seven* questions chosen from questions 1–9. Each part of A is worth 4 marks, for a total of 16, and each of 1–9 is worth 12 marks. Note that question A involves much more work per mark earned than questions 1–9. No special aids (books, notes, calculators) are to be used.

A. Please do *four* of parts (i)–(vi). If you attempt more than four, indicate clearly which four you want marked.

(i) Write a paragraph about al-Khwārizmī.

(ii) Write a paragraph about positional base 10 notation for the positive integers.

(iii) Write a paragraph about prime numbers. The paragraph should include a definition of prime number, and some additional facts and results. Do not include proofs.

(iv) Write a paragraph about perfect numbers. This should include a definition of perfect number, and some additional facts and results. Do not include proofs.

(v) Show that $\sqrt{2}$ is irrational.

(vi) Sketch a proof of the Pythagorean Theorem.

. . . .

Please do *seven* of the following nine questions. If you attempt more than seven, the best seven will count.

1. Let $u = 1001$ and let $v = 780$. (a) Use the Euclidean Algorithm to find the greatest common factor of u and v. (There are other ways to find the GCF, but credit will only be given if the Euclidean Algorithm is used.) (b) Compute explicitly the least common multiple of u and v.

2. A *bridge hand* consists of 13 cards taken from a standard 52-card deck. What is the probability that a randomly chosen bridge hand has 2 or more aces? An *expression* for the answer is enough. Please do not evaluate!

3. The figure below consists of 16 points. Each is 1 unit away from its nearest horizontal and vertical neighbours. How many triangles are there whose vertices are 3 of these 16 points? Give an explicit numerical answer.

4. How many ways are there to distribute 101 identical loonies between Alfie, Beth, and Gamal so that each gets at least one loonie? Only how much each gets matters. Give an explicit numerical answer.

5. A highly evolved species of cockroach writes numbers in what we call base 6 notation. Oddly enough, they use the "digits" 0, 1, 2, 3, 4, and 5 with their ordinary meanings. Let a have the base 6 representation 1234 and let b have the base 6 representation 5. So $a = (1234)_6$ and $b = (5)_6$. Find the base 6 representation of the product $a \times b$.

6. Let N be the 11-digit number with decimal representation $8888ddd8888$, where the middle three digits (represented by d, d, and d) are identical, maybe three 0's, maybe three 1's, maybe three 2's, and so on. For what value(s) of d does N leave a remainder of 1 on division by 9? Explain.

7. Let $N = 2^{10} \times 79$. What is the sum of the positive divisors of N? Give an explicit numerical answer.

8. There is an old "fairground" game called *blind dice.* The game is played with six special cubical dice. Each of the dice has five blank sides. The remaining sides are marked 1, 2, 3, 4, 5, and 6 respectively. You toss the dice, and your score is whatever total they show, with blanks counting as 0. What is the probability that you get a score of 2? A calculator-ready expression is enough.

9. The *Lucas numbers* L_1, L_2, L_3, L_4, and so on are defined as follows: $L_1 = 1$, $L_2 = 3$, and for all $n \geq 2$, $L_n = L_{n-1} + L_{n-2}$. So $L_3 = 4$, $L_4 = 7$, $L_5 = 11$, $L_6 = 18$, and so on. What is the smallest $n > 100$ such that L_n is a multiple of 3? Explain carefully how you *know* that your answer is correct.