The University of British Columbia. Mathematics 322

Final Examination. Friday, December 16, 2016. Instructor: Reichstein

Every problem is worth 6 points.

Name: ______________________ **Student number:** ______________

Signature: ______________________________

Rules governing examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  (a) speaking or communicating with other candidates, unless otherwise authorized;
  (b) purposely exposing written papers to the view of other candidates or imaging devices;
  (c) purposely viewing the written papers of other candidates;
  (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1	
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Total	

Problem 1: Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 9 & 5 & 1 & 8 & 6 & 4 & 7 & 10 & 3 & 2 \end{pmatrix}$

(a) Write σ as a product of disjoint cycles.

(b) Is σ even or odd?

(c) Find σ^{999}.

Problem 2: Let G be the group $C_{25} \times C_{25} \times C_5$. Here, as usual C_n, denotes the cyclic group of order n.

(a) How many elements of order 5 does G have?

(b) Is G isomorphic to $C_{25} \times C_5 \times C_5 \times C_5$? Supply a proof for your answer.

Problem 3: Let a and b be elements of a finite group G. Show that ab and ba have the same order. That is, $o(ab) = o(ba)$.

Problem 4: Let G be a finite group and H be a normal subgroup of G. Suppose the index $[G : H]$ is ≤ 5. Show that the commutator subgroup G' of G is contained in H.

Problem 5: Let $n \geqslant 5$. Show that the symmetric group S_n does not have a subgroup H of index d, for any $2 < d < n$.

Problem 6: Show that the alternating group A_4 does not have a subgroup of order 6.

Problem 7: Show that every group of order 85 is cyclic.

Problem 8: Recall that a group G is called *simple* if G has no normal subgoups, other than $\{e\}$ and G itself. Show that a group of order 105 cannot be simple.