Marks

[18] **1.** **Short answers.** Answer each question below. Read and think carefully. For this question only, no explanation or justification is needed, and no credit will be given for an incorrect answer.

(There are no typos in this question. If something looks incorrect, you should say so in your answer.)

(a) *(3 marks)* True or false? If $\mathbf{r}(t)$ is the position at time $t$ of an object moving in $\mathbf{R}^3$, and $\mathbf{r}(t)$ is twice differentiable, then $|\mathbf{r}''(t)|$ is the tangential component of its acceleration.

(b) *(3 marks)* Let $\mathbf{r}(t)$ is a smooth curve in $\mathbf{R}^3$ with unit tangent, normal and binormal vectors $\mathbf{T}(t)$, $\mathbf{N}(t)$, $\mathbf{B}(t)$. Which two of these vectors span the plane normal to the curve at $\mathbf{r}(t)$?

(c) *(3 marks)* True or false? If $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ is a vector field on $\mathbf{R}^3$ such that $P, Q, R$ have continuous first order derivatives, and if $\operatorname{curl}\mathbf{F} = \mathbf{0}$ everywhere on $\mathbf{R}^3$, then $\mathbf{F}$ is conservative.

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(d) *(3 marks)* True or false? If $\mathbf{F} = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k}$ is a vector field on $\mathbf{R}^3$ such that $P, Q, R$ have continuous second order derivatives, then $\text{curl}(\text{div}\,\mathbf{F}) = \mathbf{0}$.

(e) *(3 marks)* True or false? If $\mathbf{F}$ is a vector field on $\mathbf{R}^3$ such that $|\mathbf{F}(x, y, z)| = 1$ for all $x, y, z$, and if $S$ is the sphere $x^2 + y^2 + z^2 = 1$, then $\iint_S \mathbf{F} \cdot d\mathbf{S} = 4\pi$.

(f) *(3 marks)* True or false? Every closed surface $S$ in $\mathbf{R}^3$ is orientable. (Recall that $S$ is closed if it is the boundary of a solid region $E$.)

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[12] **2.** A curve in $\mathbf{R}^3$ is given by the vector equation $\mathbf{r}(t) = \langle 2t\cos t, 2t\sin t, \frac{t^3}{3}\rangle$.

(a) (6 marks) Find the length of the curve between $t = 0$ and $t = 2$.

(b) (6 marks) Find the parametric equations of the tangent line to the curve at $t = \pi$.

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[10] **3.** Let $C$ be the curve in $\mathbf{R}^2$ given by the graph of the function $y = \dfrac{x^3}{3}$. Let $\kappa(x)$ be the curvature of $C$ at the point $(x, x^3/3)$. Find all points where $\kappa(x)$ attains its maximal values, or else explain why such points do not exist. What are the limits of $\kappa(x)$ as $x \to \infty$ and $x \to -\infty$?

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[16] **4.** Evaluate the line integrals below. (Use any method you like.)

(a) (8 marks) $\int_C (x^2+y)dx + xdy$, where $C$ is the arc of the parabola $y = 9 - x^2$ from $(-3, 0)$ to $(3, 0)$.

(b) (8 marks) $\int_C \mathbf{F} \cdot \mathbf{n}\, ds$, where $\mathbf{F}(x, y) = 2x^2\mathbf{i} + ye^x\mathbf{j}$, $C$ is the boundary of the square $0 \le x \le 1,\ 0 \le y \le 1$, and $\mathbf{n}$ is the unit normal vector pointing outward.

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[10] **5.** Let $f$ be a function on $\mathbf{R}^3$ such that all its first order partial derivatives are continuous. Let $S$ be the surface $\{(x, y, z) : \ f(x, y, z) = c\}$ for some $c \in \mathbf{R}$. Assume that $\nabla f \neq 0$ on $S$. Let $\mathbf{F}$ be the gradient field $\mathbf{F} = \nabla f$.

(a) (4 marks) Let $C$ be a piecewise smooth curve contained in $S$ (not necessarily closed). Must it be true that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$? Explain why.

(b) (6 marks) Prove that for any vector field $\mathbf{G}$,

$$\iint_S (\mathbf{F} \times \mathbf{G}) \cdot d\mathbf{S} = 0.$$

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[10] **6.** Let $\mathbf{F} = (2y + 2)\mathbf{i}$ be a vector field on $\mathbf{R}^2$. Find an oriented curve $C$ from $(0, 0)$ to $(2, 0)$ such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 8$.

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[24] **7.** Evaluate the surface integrals. (Use any method you like.)

(a) (8 marks) $\iint_S z^2 dS$, if $S$ is the part of the cone $x^2 + y^2 = 4z^2$ where $0 \le x \le y$ and $0 \le z \le 1$.

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(b) (8 marks) $\displaystyle\iint_S \mathbf{F}\cdot d\mathbf{S}$, if $\mathbf{F} = z\mathbf{k}$ and $S$ is the rectangle with vertices $(0,2,0)$, $(0,0,4)$, $(5,2,0)$, $(5,0,4)$, oriented so that the normal vector points upward.

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(c) (8 marks) $\iint_S \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = (y - z^2)\mathbf{i} + (z - x^2)\mathbf{j} + z^2\mathbf{k}$ and $S$ is the boundary surface of the box $0 \leq x \leq 1$, $0 \leq y \leq 2$, $0 \leq z \leq 3$, with the normal vector pointing outward.

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**Be sure that this examination has 12 pages including this cover**

**The University of British Columbia**

Sessional Examinations - December 2011

**Mathematics 317**

*Advanced Calculus IV*

Closed book examination

Time: 2.5 hours

**Print Name** ____________________ **Signature** ____________________

**Student Number**____________________ **Instructor's Name** ____________________

**Section Number** ____________________

## Special Instructions:

No calculators, electronic devices (including cell phones), notes, or books of any kind are allowed.
Show all calculations for your solutions. If you need more space than is provided, use the back of the page or the additional blank page at the end of the booklet.

### Rules governing examinations



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| 1 | | 18 |
| 2 | | 12 |
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| 4 | | 16 |
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| Total | | 100 |