1. Let $\mathbf{F}$, $\mathbf{G}$ be vector fields, and f, g be scalar fields. Assume $\mathbf{F}$, $\mathbf{G}$, f, g are defined on all of $\mathbb{R}^3$ and have continuous partial derivatives of all orders everywhere.
Mark each of the following as True (T) or False (F). No reason or justification is required.

(1) If C is a closed curve and $\text{grad}\, f = \mathbf{0}$ then $\int_C f ds = 0$.

(2) If $\mathbf{r}(t)$ is a parametrization of a smooth curve C, and the binormal $\mathbf{B}(t)$ is constant, then C is a straight line.

(3) If $\mathbf{r}(t)$ is the position of a particle which travels with constant speed then $\mathbf{r}'.\mathbf{r}'' = 0$.

(4) If C is a path from points A to B then the line integral $\int_C (\mathbf{F} \times \mathbf{G}).d\mathbf{r}$ is independent of the path C.

(5) The line integral $\int_C f ds$ does not depend on the orientation of the curve C.

(6) $\iint_S (\text{curl}\, \mathbf{F}.\mathbf{n}) dS = 0$ for every closed surface S if and only if $\mathbf{F}$ is conservative.

(7) If S is a parametric surface $\mathbf{r}(u, v)$ then a normal to S is given by

$$\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial u}.$$

(8) The surface area of the parametric surface S given by $\mathbf{r}(u, v) = x(u, v)\mathbf{i} + y(u, v)\mathbf{j} + z(u, v)\mathbf{k}$, $(u, v) \in D$, is given by

$$\iint_D \left(1 + \left(\frac{\partial z}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2\right)^{1/2} dA.$$

(9) If $\mathbf{F}$ is the velocity field of an incompressible fluid then $\text{div}\, \mathbf{F} = 0$.

(10) $\text{div}(\mathbf{F} \times \mathbf{G}) = (\text{div}\, \mathbf{F})\mathbf{G} + (\text{div}\, \mathbf{G})\mathbf{F}$.

2. Let C be the space curve

$$\mathbf{r}(t) = (e^t - e^{-t})\mathbf{i} + (e^t + e^{-t})\mathbf{j} + 2t\mathbf{k}.$$

(a) Find $\mathbf{r}'$, $\mathbf{r}''$ and the curvature of C.

(b) Find the length of the curve between $\mathbf{r}(0)$ and $\mathbf{r}(1)$.

3. Let D be the domain consisting of all (x, y) such that $x > 1$, and let $\mathbf{F}$ be the vector field

$$\mathbf{F}(x, y) = -\frac{y}{x^2 + y^2}\mathbf{i} + \frac{x}{x^2 + y^2}\mathbf{j}.$$

Is $\mathbf{F}$ conservative on D? Give reasons for your answer.

4. Let

$$\mathbf{F}(x,y) = (\tfrac{3}{2}y^2 + e^{-y} + \sin x)\mathbf{i} + (\tfrac{1}{2}x^2 + x - xe^{-y})\mathbf{j}.$$

Find $\int_C \mathbf{F}.d\mathbf{r}$, where C is the boundary of the triangle $(0,0)$, $(1,-2)$, $(1,2)$, oriented anticlockwise.

5. Let S be the part of the paraboloid $z = 1 - x^2 - y^2$ lying above the xy plane. At (x, y, z) S has density

$$\rho(x, y, z) = \frac{z}{(5 - 4z)^{1/2}}.$$

Find the centre of mass of S.

6. Evaluate $\iint_S (\text{curl}\,\mathbf{F}.\mathbf{n})dS$ where S is that part of the sphere $x^2 + y^2 + z^2 = 2$ above the plane $z = 1$, $\mathbf{n}$ is the upward unit normal, and

$$\mathbf{F}(x, y, z) = -y^2\mathbf{i} + x^3\mathbf{j} + (e^x + e^y + z)\mathbf{k}.$$

7. Let B be the solid region lying between the planes $x = -1$, $x = 1$, $y = 0$, $y = 2$ and bounded below by the plane $z = 0$ and above by the plane $z + y = 3$. Let S be the surface of B. Find the flux of the vector field

$$\mathbf{F}(x, y, z) = (x^2 z + \cos \pi y)\mathbf{i} + (yz + \sin \pi z)\mathbf{j} + (x - y^2)\mathbf{k}$$

outwards through S.