Be sure that this examination has 7 pages, including this cover.

The University of British Columbia
Final Examinations – December 2009
Mathematics 312
Instructor: V. Vatsal
Time: 2.5 hours

Name:
Student Number:

Signature:

Special instructions:

1. No calculators, books, notes, or other aids allowed.
2. Answer all 6 questions. Each question is worth 10 points, for a total of 60 points.
3. Give your answer in the space provided. If you need extra space, use the back of the page.
4. Show enough of your work to justify your answer. Show ALL steps.

Rules governing examinations:

1. Each candidate should be prepared to produce his/her library/AMS card upon request.
2. No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in the examination questions.
3. Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination, and and shall be liable to disciplinary action:

   a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

   b) Speaking or communicating with other candidates.

   c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness will not be received.

Problem 1:

a) Show that $(3!)^n$ divides $(3n)!$, for every positive integer n.

b) Suppose that a, b are integers with $(a, b) = 1$. Then show that $(a+b, a^2+b^2)$ is either 1 or 2.

Problem 2:

a) Determine whether 209 passes Miller's test to the base 2.

b) Determine whether 2821 is a Carmichael number. Explain your answer.

Problem 3:

a) Find the last three decimal digits of the number 7^{999}.

b) Find $9^{23} \bmod 31$.

Problem 4:

a) Decrypt OAPB, which was encrypted by the affine transformation $C \equiv 7P + 11 \bmod 26$.

b) Suppose that p is an odd prime. Then show that $1^2 \cdot 3^2 \cdot 5^2 \cdot (p-2)^2 \equiv (-1)^{(p+1)/2} \bmod p$.

Problem 5:

a) Let μ denote the Möbius function. If $n > 1$ is an integer, show that $\sum_{d|n} \mu(d) = 0$.

b) Compute the convergents of the continued fraction $[0; 1, 2, 3, 4]$.

Problem 6:

a) Show that $\sqrt{3} + \sqrt{2}$ is irrational.

b) For any positive integers a and n, show that $a \equiv a^{4n+1} \bmod 10$.