

The University of British Columbia

Final Examination - April 17, 2009

Mathematics 308

Section 201 – D. Rolfsen, Instructor

Closed book examination

Time: 2.5 hours

Last Name _____ First _____ Signature _____

Student Number _____

Special Instructions:

No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in “calculator-ready” form, where calculator means basic scientific calculator. Show all your work, little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBC-card for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
 - (b) Speaking or communicating with other candidates.
 - (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		6
2		10
3		24
4		8
5		8
6		6
7		6
8		10
9		10
10		6
11		6
Total		100

Marks
[6]

1. What is the interior angle of a regular 7-gon, measured in radians?

Answer:

[10]

2. (a) Write the complex number $5e^{-3\pi i/4}$ in the form $x + iy$:

Answer:

- (b) Give all complex solutions Z to the equation $Z^2 = 1 - i$. (You may write your answers in polar form or the form $x + iy$.)

Answer:

3. True or false? Explain.

[3] (a) For any two vectors X and Y in \mathbb{R}^n we have $|X| + |Y| \leq |X - Y|$.

True/False?

[3] (b) In any triangle, the centroid, circumcentre, and orthocentre are collinear.

True/False?

[3] (c) The inequality $|x| + |y| + |z| \leq 1$ defines a regular octahedron in \mathbb{R}^3 .

True/False?

[3] (d) The circumcentre of a triangle lies outside the triangle if one of the triangle's angles is greater than a right angle.

True/False?

[3]

- (e) If four distinct complex numbers do not lie in a straight line or circle, then their cross-ratio cannot be real.

[3]

- (f) There are exactly 48 isometries of the regular dodecahedron (including the identity).

[3]

- (g) If Q and Q' are quaternions, the conjugate of QQ' is equal to $\bar{Q}\bar{Q}'$.

[3]

- (h) If Q is a pure imaginary quaternion and $Q \neq 0$, then Q^{-1} is also pure imaginary.

[8]

4. Consider the triangle ABC in \mathbb{R}^2 whose vertices A, B, C have coordinates $(0, 1), (t, 0)$ and $(s, 0)$, respectively.

(a) Denote the medians of ABC , as usual, by AA', BB' and CC' . Find the coordinates of A' and of the centroid G .

 $A' =$ $G =$

(b) Let the altitudes of ABC be denoted AD, BE and CF , respectively. Find the coordinates of D and of the orthocentre H .

 $D =$ $H =$

[8] 5. Consider the Möbius transformation $T(Z) = \frac{iZ}{1+Z}$.

(a) What is the image of the unit circle under T ?

Answer:

(b) What points are fixed by T ?

Answer:

(c) Give a formula for the inverse transformation T^{-1} (meaning $T^{-1}(T(Z)) = Z$)

Answer:

(d) What is the *inverse* image of the unit disk under T ?

Answer:

- [6] 6. Find a Möbius transformation $T(Z) = \frac{AZ + B}{CZ + D}$ satisfying:

$$T(0) = 0, \quad T(1) = 1, \quad T(i) = \infty.$$

Answer:

- [6] 7. Suppose C and C' are intersecting circles in the complex plane, and R and R' are inversions (a.k.a. reflections) in C and C' , respectively. If $R(C') = C'$, does it follow that $R'(C) = C'$? Explain.

Answer:

[10]

8. The quaternion $Q = 5 + 4i - 3k$ has a polar form $Q = r(\cos \theta + H \sin \theta)$, where $H^2 = -1$. Determine the value of r , θ and H .

$$r =$$

$$\theta =$$

$$H =$$

Calculate Q^2 and the inverse of Q .

$$Q^2 =$$

$$Q^{-1} =$$

- [10] 9. If Q is a unit quaternion, define $M_Q(X) = QX\bar{Q}$, where X is a quaternion variable.
- (a) Find the value of Q so that the action of M_Q on the imaginary quaternions

$$\mathbb{R}^3 = \{xi + yj + zk\}$$

consists of $\pi/3$ rotation about the line defined by the equations $x = z, y = 0$.

$Q =$

- (b) If $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is the rotation by π about the z -axis, find the unit quaternion Q' so that $M_{Q'}(X) = T(M_Q(X))$ for all $X \in \mathbb{R}^3$

$Q' =$

- [6] 10. Suppose $R: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is reflection in a plane P and that $R(0, 0, 0) = (1, 2, 3)$. Find an equation for P :

Answer:

- [6] 11. Let the subset S of $\mathbb{R}^4 = \{(x_1, x_2, x_3, x_4)\}$ be defined by the conditions:

$$x_i \geq 0 \text{ for } i = 1, 2, 3, 4 \quad \text{and} \quad x_1 + x_2 + x_3 + x_4 = 1.$$

Then S is a regular 3-dimensional polyhedron. Identify the polyhedron and calculate the length of its edges.

Polyhedron name:

Edge length =