Marks

[10] **1.**

(a) [5] Compute the LU decomposition of the matrix $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 3 & 4 & 1 \\ 0 & -1 & 1 & 1 \end{bmatrix}$

(b) [5] If $\mathbf{x}$ solves the equation $A\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, find $U\mathbf{x}$.

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[10] **2.**

(a) [5] Compute the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$. What are the dimensions of the nullspace, the column space, the row space, and the left nullspace?

(b) [5] Find an orthogonal basis for the space spanned by $\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 4 \\ 5 \end{bmatrix}$ in R^4.

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[21] **3.** Decide whether each of the following statements is true or false. You need not give a reason. All matrices in this question are square ($n \times n$).

(a) [3] If $A = A^{-1}$ then every eigenvalue of A is either 1 or -1.

(b) [3] The diagonal entries of an upper triangular matrix are its eigenvalues.

(c) [3] Eigenvalues of an anti-symmetric matrix (i.e., $A^T = -A$) are negative.

(d) [3] If S is any invertible matrix, then A and SAS^{-1} have the same determinant.

(e) [3] If U and V are orthogonal and $A = U\Sigma V^T$ then A and Σ have the same eigenvalues.

(f) [3] If 2 is an eigenvalue of A then 6 is an eigenvalue of $A + A^2$.

(g) [3] If $A = LU$ is the LU decomposition of A, then A and U have the same eigenvalues.

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[15] **4.**

(a) [10] Show that $e^{A+B} \neq e^A e^B$ for $A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$. (Hint: $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$)

(b) [5] Let A and B be any two $n \times n$ matrices. Show that if λ is an eigenvalue of AB and $\lambda \neq 0$ then λ is also an eigenvalue of BA.

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[12] **5.** Let X be the subspace in three dimensional space $\mathbb{R}^3$ containing all vectors perpendicular to $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$. Let L be the linear transformation defined by first projecting a vector onto the x–y plane and then projecting the resulting vector onto X.

(a) [4] Show that the vectors $\mathbf{v_1} = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$ and $\mathbf{v_2} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ form a basis for X.

(b) [4] Find the 3×3 matrix the represents L as a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$

(c) [4] If $\mathbf{x}$ lies in X then so does $L(\mathbf{x})$. This means that L defines a linear transformation from X to X. Find the matrix of this transformation with repsect to the basis in part (a).

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[10] **6.** Consider the differential equation $\mathbf{x}'(t) = \begin{bmatrix} -1 & -\alpha \\ 1 & -1 \end{bmatrix} \mathbf{x}(t)$.

(a) [5] For what values of α is the system stable, neutrally stable or unstable?

(b) [5] Find the matrix exponential e^{tA} when $\alpha = 4$

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[10] **7.** The equation

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

has no solution.

(a) [7] Find the “least squares” solution.

(b) [3] Explain in what way it is the best possible approximation to a solution.

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[12] **8.**

(a) [4] Find the 3 by 3 matrix A with eigensystem $\lambda_1 = 1, \lambda_2 = 2/3, \lambda_3 = 0$ and $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ -1/\sqrt{2} \end{bmatrix}$.

(b) [4] Calculate $A^{100} \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}$, up to negligible error.

(c) [4] What is the entry in the second row and second column of A^{100}?

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The University of British Columbia
Sessional Examinations - December 2005

Mathematics 307
Applied Linear Algebra

Closed book examination Time: 2.5 hours

Print Name ______________ **Signature** ______________

Student Number______________ **Instructor's Name** ______________

Section Number ______________

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Total		100