Be sure this exam has 11 pages including the cover

The University of British Columbia

MATH 305, Sections 201

Final Examination, April 27, 2017, 2.5 hours

Name ______________________________ **Signature** ______________________________

Student Number ____________________

This exam consists of **9** questions. No notes. Write your answer in the blank page provided.

Problem	max score	score
1.	12	
2.	8	
3.	10	
4.	10	
5.	10	
6.	8	
7.	8	
8.	16	
9.	18	
total	100	

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

(12 points) 1. Find all solutions to

$$(a) \quad e^{z^2} = -1 \qquad\qquad (b) \quad \sqrt{z} = 1 - i$$

where $\sqrt{z}$ denotes the principal branch.

(8 points) 2. Let $f(z) = y^3 - 3x^2y - 3y + i(-x^3 - 3xy^2 + 3x)$ where $z = x + iy$. Where is f differentiable in the complex plane? Where is f analytic? Explain your reasoning carefully.

(10 points) 3. Find an analytic mapping from $\{(x, y) \mid 1 \leq x \leq 3,\ y \geq 0\}$ into the upper half plane $\{(u, v) |\ v \geq 0\}$ so that $(1, 0)$ is mapped to $(-1, 0)$ and $(3, 0)$ is mapped to $(1, 0)$.

Hint: consider the map $\sin(z)$.

(10 points) 4. Find a branch for $f(z) = (z^2 - 1)^{\frac{1}{2}}$ so that f is analytic in $C\backslash((-\infty, -1] \cup [1, +\infty))$ and $f(0) = -i$.

(10 points) 5. Find the solution to

$$u_{xx} + u_{yy} = 0 \ \text{ in } \{(x, y) \mid y > 0\}$$

$$u(x,0) = \begin{cases} -1 & \text{for } x < -1; \\ 2 & \text{for } -1 < x < 2; \\ 0 & \text{for } x > 2 \end{cases}$$

(8 points) 6. Recall Liouville's Theorem: all bounded entire (analytic) functions are constants. Use it to prove that there are no nonconstant entire function $f(z)$ such that $Im(f(z)) \geq 0$.

(8 points) 7. Use Rouche's theorem to find the number of roots for $f(z) = 2z^5 + 5z^3 + z + 1$ in the region $\{1 < |z| < 2\}$.

(16 points) 8. Compute the residues at the given singularities:

$(a)\ z^3\cos(\frac{1}{z}),\ z_0 = 0 \qquad (b)\ \frac{z}{\text{Log}(z)},\ z_0 = 1, \qquad (c)\ \frac{e^z}{z\sin z},\ z_0 = 0 \qquad (d)\ \frac{e^z}{(z-1)^5},\ z_0 = 1$

where Log (z) denotes the principal branch.

(18 points) 9. By using Cauchy residue theory, calculate the following integrals. Justify your answers.

$$(a)\ \int_0^{\infty} \frac{\sqrt{x}}{x^2+2x+2}dx, \qquad (b)\ \int_{-\infty}^{+\infty} \frac{\sin(x)}{x^2+2x+2}dx$$