The University of British Columbia. Mathematics 300

Final Examination. Thursday, April 13, 2017. Instructor: Reichstein

Name: ______________________________ **Student number:** ____________________

Signature: ____

Rules governing examinations

• Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.

• Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

• No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.

• Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

• Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:

(a) speaking or communicating with other candidates, unless otherwise authorized;

(b) purposely exposing written papers to the view of other candidates or imaging devices;

(c) purposely viewing the written papers of other candidates;

(d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,

(e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

• Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

• Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.

• Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

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Problem 1: (8 marks) Determine whether the following statements are TRUE or FALSE. In each part supply a brief explanation to justify your answer.

(a) $|\cos(z)| \leq 1$ for all z in the upper half-plane.

(b) The function $f(z) = z^2 + 1$ maps the circle $|z| = 2$ onto another circle.

(c) The function $f(x + \mathrm{i}y) = (x^2 - y^2 + y) + \mathrm{i}(2xy + x)$ is analytic in the entire plane.

(d) $\sin(\bar{z}) = \overline{\sin(z)}$ for every complex number z. Here $\bar{w}$ is the complex conjugate of w.

Problem 2: (6 marks) Find all complex numbers z satifying $z^8 - 3z^4 - 4 = 0$.

Problem 3: (6 marks) Evaluate the integral

$$\int_\Gamma \frac{\mathrm{Log}(1-z)}{(1+z)^n}\,\mathrm{d}z\,,$$

where Γ is the positively oriented circle $|z+1|=1$ and n is an arbitrary integer.
Hint: Consider the cases, where $n \leq 0$, $n=1$, and $n \geq 2$ separately.

Problem 4: (6 marks) Evaluate the following integrals.

(a) $\displaystyle\int_\Gamma \frac{z+1}{z^2(z^2-4)}\,\mathrm{d}z$, where Γ is the positively oriented circle $|z-1|=2$.

(b) $\displaystyle\int_\Gamma (z^2+1)\sin\left(\frac{2}{z}\right)\mathrm{d}z$, where Γ is the positively oriented unit circle $|z|=1$.

Problem 5: (6 marks) Find the radius of convergence of the power series $\sum_{n=0}^{\infty} 3^{-n} n^2 z^n$.

Problem 6: (6 marks) Suppose $f(z)$ is an entire function. As usual, we will denote the derivative of $f(z)$ by $f'(z)$. If we know that

$$f'\left(\frac{1}{n}\right) = \frac{1}{n}$$

for every positive integer n, and $f(0) = 1$, what is $f(2\mathrm{i})$?

Problem 7: (6 marks) Find all zeros and singularities of the function

$$f(z) = \frac{z^3 e^z \sin(z)}{(1-\cos(z))^2}$$

in the complex plane. For each singularity determine whether it is removable, essential or a pole. Find the order of each zero and of each pole.

Problem 8: (6 marks) Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{x^6+8}\,\mathrm{d}x$.