FINAL EXAM

Math 300, April 24, 2009

Last Name: ______________________________

First Name: ______________________________

Student Number: ______________________________

Section Number: ______________________________

Signature: ______________________________

The exam is worth a total of 100 points with duration 2.5 hours. **No books, notes, formula sheets or calculators are allowed**. Justify all answers, show all work and **explain your reasoning carefully**. You will be graded on the clarity of your explanations as well as the correctness of your answers.

UBC Rules governing examinations:

(1) Each candidate should be prepared to produce his/her library/AMS card upon request.
(2) No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in the examination questions.
(3) Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination, and and shall be liable to disciplinary action:
a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
b) Speaking or communicating with other candidates.
c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness will not be received.
(4) Smoking is not permitted during examinations.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Problem 1 (14 points). Let $u(x, y) = (e^y + e^{-y}) \cos x + (e^y - e^{-y}) \sin x$ for $x, y \in \mathbb{R}$.

(a) Verify that u is harmonic.

(b) Find an analytic function f such that $Re f = u$ and $f(0) = 2$.

(c) Express the function f found in (b) in the complex variable z.

Problem 2 (12 points). (a) Find the principal value of $\left(\frac{e}{2}(-1-\sqrt{3}\,i)\right)^{3\pi i}$.

(b) Find $Res\left(\dfrac{\operatorname{Log} z}{(z-i)^3}; i\right)$, where $\operatorname{Log} z$ is the principal branch of $\log z$.

Problem 3 (8 points). Let γ be the semi-circle with a given direction as shown in the figure below. Evaluate the line integral $\int_\gamma \frac{1+z}{z^2}\, dz$.

Problem 4 (14 points). Find the Laurent series of $\frac{1}{z(z-1)(z-2)}$ in the two regions respectively.

(a) $0 < |z| < 1$

(b) $1 < |z| < 2$

Problem 5 (12 points). Find and classify all singularities of the following functions. You need to provide reasonings.

(a) $z^2 \sin \dfrac{1}{z}$

(b) $\dfrac{e^{-z} \sin^2 z}{z^2(1+z^2)^3}$

Problem 6 (14 points). Evaluate the integral $\int_\gamma \frac{e^z}{z^2(z-1)}\,dz$ where γ is shown in the figure below.

Problem 7 (14 points). Use the residue theory to evaluate $p.v. \int_{-\infty}^{\infty} \frac{3x^3}{(1+x^2)(4+x^2)}\, dx.$

Problem 8 (12 points). If u is a harmonic function on $\mathbb{R}^2$ and $u \geq 0$, show that u is constant.