FINAL EXAM

Math 300, December 17, 2008

Last Name: ______________________________

First Name: ______________________________

Student Number: ______________________________

Signature: ______________________________

The exam is worth a total of 100 points with duration 2.5 hours. No books, notes or calculators are allowed. Justify all answers, show all work and **explain your reasoning carefully**. You will be graded on the clarity of your explanations as well as the correctness of your answers.

UBC Rules governing examinations:

(1) Each candidate should be prepared to produce his/her library/AMS card upon request.

(2) No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in the examination questions.

(3) Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination, and and shall be liable to disciplinary action:

a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

b) Speaking or communicating with other candidates.

c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness will not be received.

(4) Smoking is not permitted during examinations.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Problem 1 (10 points, 1 pt. each) For each of the following statements, circle T if the statement is true, and F if the statement is false.

T F If $f(z)$ satisfies the Cauchy-Riemann equations at z_0, then $f(z)$ is differentiable at z_0.

T F If $f(z)$ has a pole at z_0, then $\lim_{z \to z_0} |f(z)| = \infty$.

T F If $f(z)$ is analytic in a domain D containing a simple closed contour Γ, then $\int_\Gamma f(z)\, dz = 0$.

T F If the two power series $\sum_{k=0}^{\infty} a_k (z - z_0)^k$ and $\sum_{k=0}^{\infty} b_k (z - z_0)^k$ converge to the same function in the disk $\{|z - z_0| = 1\}$, then $a_k = b_k$ for all k.

T F There does not exist any function $f(z)$ which is analytic at the point 0 and nonanalytic everywhere else.

T F The function $\text{Log}(z^2)$ is analytic for all values of z except those on the negative real axis.

T F Any entire function is the complex derivative of another entire function.

T F If $f(z)$ has an essential singularity at z_0, then $\text{Res}((z - z_0)f(z); z_0) = 0$.

T F If $f(z)$ and $g(z)$ have simple poles at 0, then $(fg)(z)$ has a simple pole at 0.

T F If the disk of convergence of the Taylor series of a function $f(z)$ is $\{|z| = 2\}$, then the disk of convergence for the Taylor series of $f(z^2)$ is $\{|z| = 4\}$.

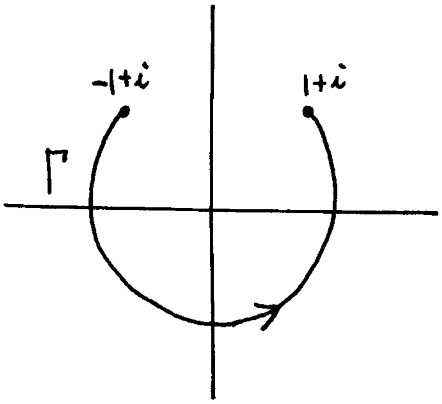
Problem 2 (10 points) Use the LIMIT DEFINITION of differentiability to show that for any $z_0 \neq 0$, $f(z) = \overline{z}^2$ is not differentiable at z_0.

Problem 3 (10 points) Suppose that the functions $f(z)$ and $g(z)$ are entire, $|f(z)| < |g(z)|$ for all z, and $f(0) \neq 0$. Show that $f(z)$ has no zeros. (In other words, show that $f(z) \neq 0$ for all z.)

Problem 4 (12 points) Find

$$\int_\Gamma \frac{1}{z}\, dz,$$

where Γ is the portion of the circle $\{|z| = \sqrt{2}\}$, traversed counterclockwise, beginning at $-1+i$, and ending at $1+i$.



Problem 5 (8 points) Let $f(z)$ be a function analytic on the disk $\{|z| < 1\}$. Show that it cannot be true that $|f^{(k)}(0)| \geq k!2^k$ for all nonnegative integers k.

Problem 6 (12 points) Find the Laurent series centered at $z_0 = 0$ of

$$f(z) = \frac{z}{z^2 - 9}$$

in the following domains.

(a) $D = \{|z| < 3\}$
(b) $D = \{|z| > 3\}$

Problem 7 (12 points) Classify all isolated singularities of

$$f(z) = \frac{z^2(z-1)}{\sin^2(\pi z)}.$$

Problem 8 (13 points) Find

$$\int_C \frac{e^{\pi z}}{(z^2+1)(z-3i)^2}\, dz,$$

where C is the negatively oriented circle $\{|z|=2\}$.

Problem 9 (13 points) Using RESIDUE THEORY, evaluate

$$\text{p.v.} \int_{-\infty}^{\infty} \frac{x^2 + x}{(x^2 + 1)^2}\, dx.$$

(In other words, find $\lim_{\rho \to \infty} \int_{-\rho}^{\rho} \frac{x^2+x}{(x^2+1)^2}\, dx$.)