MATHEMATICS 300 FINAL EXAM

APRIL 27, 2006. INSTRUCTORS: D. SJERVE, Z. REICHSTEIN

This is a closed book exam. You can use one 8.5” × 11” note sheet but no books or calculators are allowed. In order to receive credit for a problem you need to show enough work to justify your answer.

Name (Please print):

Student number:

Problem	**Score**	**Problem**	**Score**
1		5	
2		6	
3		7	
4		8	

TOTAL

(6 marks) **Problem 1:** Find all complex solutions to the equation $\cos(z) = 2i\sin(z)$. Express each solution in the form $z = x + yi$, where x and y are real numbers.

(6 marks) **Problem 2:** Answer true or false to the following statements. Give valid reasons for all your answers.

(a) $\operatorname{Log}(z^2) = 2\operatorname{Log}(z)$ for every complex number z. Here $\operatorname{Log}(z)$ denotes the principal value of $\log(z)$.

(b) If $f(z) = u(x,y) + \imath v(x,y)$ is an analytic function of $z = x + \imath y$, where $u(x,y)$ and $v(x,y)$ are real valued functions, then the function $w(x,y) = u(x,y) + v(x,y)$ is harmonic.

(c) Suppose $f(z)$ is an analytic function at $z = z_0$ and $f(z_0) = 0$. If $f(z)$ is not identically zero in any disc centered at z_0 then $g(z) = \dfrac{f'(z)}{f(z)}$ has a simple pole at z_0.

(6 marks) **Problem 3:** Find all singularities of the function $f(z) = \dfrac{z}{1-\cos(z^2)}$. Determine the nature of each singularity (i.e., whether it is removable, essential or a pole). For each pole, determine its order.

(7 marks) **Problem 4:** Prove that the function $f(z) = \bar{z}^{1000}$ is not analytic in any open disc. Here, as usual, $\overline{z}$ denotes the complex conjugate of z.

(7 marks) **Problem 5:** Suppose $f(z)$ is an entire function such that $|f(z)| < 2|z| + 3$ for all complex numbers z. Show that $f(z)$ is a polynomial of degree ≤ 1.

(6 marks) **Problem 6:** Suppose the Laurent series for the function $f(z) = \dfrac{z}{(z-1)(z-2)}$ in the annulus $1 < |z| < 2$ is given by $\sum_{j=-\infty}^{\infty} c_j z^j$. Find (a) c_{100}, (b) c_{-100}.

(6 marks) **Problem 7:** Evaluate $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 - 3\cos(\theta)}$.

(6 marks) **Problem 8:** Evaluate $\displaystyle\int_0^\infty \frac{dx}{(x^2+1)^3}$.