Math 267 Final Exam
Apr 23, 2010
Duration: 150 minutes

Name: ____________________ Student Number: ____________________

Section: ____________________

Do not open this test until instructed to do so! This exam should have 17 pages, including this cover sheet. No textbooks, calculators, or other aids are allowed. One page of notes is allowed. Turn off any cell phones, pagers, etc. that could make noise during the exam. You must remain in this room until you have finished the exam. **Circle your solutions! Reduce your answer as much as possible. Explain your work.** Relax. Use the back of the page if necessary.

Read these UBC rules governing examinations:

(i) Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.

(ii) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

(iii) No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

(iv) Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

- Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
- Speaking or communicating with other candidates.
- Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

(v) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Problem	**Out of**	**Score**
1	18	
2	12	
3	10	
4	10	
5	10	
6	15	
7	10	
8	15	
Total	100	

Problem 1 (18 points)

(a) Calculate $f(t) = (\mathrm{rect} * \sin)(t)$.

(b) Calculate the Fourier transform of $g(t) = \mathrm{rect}(3t)\cos(17t)$.

(c) Find the inverse Fourier transform of $\widehat{h}(\omega) = \dfrac{2i\omega}{(1+i\omega)(5+i\omega)}$.

(d) Calculate $\displaystyle\int_{-\infty}^{\infty} [\mathrm{sinc}(\omega/4)]^2 d\omega$.

Problem 2 (12 points)

Let $f(t)$ be a 2π-periodic function and $f(t) = t^2$ for $-\pi < t \leq \pi$.

(a) Plot $f(t)$ in at least three periods. Is $f(t)$ odd or even?

(b) Find the **real** Fourier Series of $f(t)$.

(c) Use (b) to calculate the infinite sum $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$.

Problem 3 (10 points)

Suppose an LTI system has an output $y(t) = 2\cos(5t)$ when its input is equal to $x(t) = 3\cos(5t) + 2\cos(35t) + \cos(72t + \pi/2)$.

(a) Calculate the Fourier transforms of $x(t)$ and $y(t)$.

(b) Specify a possible transfer function for this LTI system. Does this system correspond to a low-pass, high-pass, or band-pass filter?

Problem 4 (10 points)

Find the Fourier transform (with respect to x) $\hat{u}(k,t)$ of the solution $u(x,t)$ of the equation (Schrödinger equation)

$$\begin{cases} iu_t(x,t) = u_{xx}(x,t), & x \in \mathbb{R},\ t \geq 0, \\ u(x,0) = e^{-|x|}, & x \in \mathbb{R}, \\ u(-\infty,t) = u(\infty,t) = 0, & t \geq 0. \end{cases}$$

Hint: Recall that $e^{-|x|} \xleftrightarrow{\text{FT}} \dfrac{2}{1+k^2}$.

Problem 5 (10 points)

(a) Find the discrete Fourier transform of the vector

$$v = [1, 1, 1, 1, 1, 1].$$

(b) Let $x[n] = \sin(\frac{2\pi}{11}n)$.

  (a) What is the fundamental period of $x[n]$?

  (b) Find the discrete time Fourier series coefficients of $x[n]$.

Problem 6 (15 points)

Determine the z-transform of the following signals. **Specify the region of convergence in each case.**

(a) $x[n] = \left(\frac{1}{2}\right)^n u(n)$,

(b) $y[n] = \left(\frac{1}{2}\right)^n u(-n-1)$,

(c) $y[n] = \left(\frac{1}{2}\right)^{|n|}$.

(d) In each case above, indicate whether the discrete-time Fourier transform of the signal exists. If it does, find it. If not, explain.

Problem 7 (10 points)

Calculate the inverse z transforms of the following functions.

(a) $H(z) = \dfrac{z}{(z-\frac{1}{2})(z-4)}, \;\; \frac{1}{2} < |z| < 4.$

(b) $G(z) = \dfrac{z^5+1}{z-\frac{1}{3}}, \;\; \frac{1}{3} < |z|.$

Problem 8 (15 points)

A **causal** LTI system is described by the difference equation

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = x[n] + \frac{1}{3}x[n-1].$$

(a) Find the system function $H(z)$ of the system. Specify its region of convergence.

(b) Find the impulse response $h[n]$ of this system.

(c) Is this system stable? Justify your answer.

(d) Find $y[n]$ if $x[n] = \delta[n-3]$.