Be sure this exam has 11 pages including the cover

The University of British Columbia

Final Exam – December 2009
Mathematics 267, Mathematical Methods for EE and CS Students

Name ______________________________ **Signature** ______________________________

Student Number ____________________

This exam consists of **5** questions worth **100** marks in total. No notes, calculators aids are permitted.

Problem	max score	score
1.	20	
2.	20	
3.	20	
4.	20	
5.	20	
total	100	

(20 points) 1. Find the solution $u(x,t)$ of the wave equation:

$$\begin{cases} u_{tt}(x,t) = \frac{1}{\pi^2} u_{xx}(x,t), & 0 \le x \le 1, \quad t > 0, \\ u_t(x,0) = 0, & 0 \le x \le 1, \\ u(x,0) = f(x), & 0 \le x \le 1, \\ u(0,t) = u(1,t) = 0, & t \ge 0. \end{cases}$$

Here

$$f(x) = \begin{cases} \frac{3}{10}x, & \text{if} \quad 0 \le x \le \frac{1}{3}, \\ \frac{3(1-x)}{20}, & \text{if} \quad \frac{1}{3} \le x \le 1. \end{cases}$$

(20 points) 2. Let $g(x)$ be the 2π-periodic triangle wave and

$$g(x) = \begin{cases} \pi + x, & \text{if} \quad -\pi \le x \le 0, \\ \pi - x, & \text{if} \quad 0 \le x \le \pi. \end{cases}$$

(a) Plot the graph of $g(x)$ in at least three period.

(b) Find the real Fourier series of $g(x)$.

(c) Find

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots .$$

(20 points) 3. For the following questions, fill in the answers in the boxes. No work need to be shown and no partial credit will be given. **Anything written outside of the boxes is ignored.**

(a) Find the fundamental period of the signal $x[n] = 3 + \cos(\frac{5\pi n}{7} + \frac{\pi}{3})$.

Answer = $\boxed{}$

(b) Find the discrete Fourier transform of $x[n] = (\frac{1}{4})^n u(n-3)$.

Answer = $\boxed{}$

(c) Find the Fourier transform of $f(t) = te^{-3t}u(t-3)$.

Answer = $\boxed{}$

(d) Find the z-transform of $x[n] = \delta[n] + 2^n u[-n]$.

Answer = $\boxed{}$

(e) Find the inverse Fourier transform of $\hat{f}(\omega) = \frac{1}{1+\omega^2}$.

Answer = $\boxed{}$

(20 points) 4. Consider the system with input $x(t)$ and output $y(t)$ which is characterized by the ODE

$$y''(t) + 5y'(t) + 4y(t) = 3x(t).$$

(a) Find the transfer function $\hat{H}(\omega)$ and the impulse response $H(t)$.

(b) Find $y(t)$ if $x(t) = \delta(t-4)$.

(c) Find $y(t)$ if $x(t) = e^{-4t}u(t-2)$.

(20 points) 5. Assume that $H(z)$ is the z-transform of the discrete signal $h[n]$ and

$$H(z) = \frac{z^2 - 3z}{z^2 - \frac{3}{2}z + 1}.$$

(a) Find the regions of convergence so that $H(z)$ is causal. Find $h[n]$ is this case.

(b) Find the regions of convergence so that $H(z)$ is stable. Find $h[n]$ is this case.