Be sure that this examination has 12 pages including this cover

The University of British Columbia

Final Examinations - April, 2005

Mathematics 267

Mathematical Methods for Electrical and Computer Engineering

Closed book examination

Time: $2\frac{1}{2}$ hours

Name ____________________ **Signature** ____________________

Student Number____________________ **Instructor's Name** ____________________

Section Number ____________________

Special Instructions:

To receive full credit, all answers must be supported by clear and correct derivations.
No calculators, notes, or other aids are allowed. A formula sheet is provided with the exam.

Rules Governing Formal Examinations

1. Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.

2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

3. No candidate shall be permitted to enter the examination room after the expiration of one half hour from the scheduled starting time, or to leave during the first half hour of the examination.

4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

1		15
2		15
3		15
4		15
5		15
6		10
7		15
Total		100

Marks

[15] **1.** Let $L > 1$ and consider the pulse function (boxcar)

$$b(t) = \begin{cases} 1, & |t| < 1 \\ 0, & 1 < |t| < L \end{cases}, \qquad b(t+2L) = b(t)$$

(a) Find the Fourier series for $b(t)$. Be careful with the constant term.

(b) Use Parseval's theorem and your answer to part (a) to find the value of

$$S = 1 + \tfrac{1}{9} + \tfrac{1}{25} + \tfrac{1}{49} + \cdots = \sum_{k=1}^{\infty} \tfrac{1}{(2k-1)^2}$$

Hint: Choose $L = 2$.

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[15] **2.** Consider the following boundary-value problem:

$$u_{tt}(x,t) + u_{xx}(x,t) = 0 \qquad \text{(PDE)}$$
$$u(0,t) = u(\pi,t) = 0 \qquad \text{(BC)}$$
$$u_t(x,0) = 0 \text{ and } u_t(x,1) = 2\sin x + 6\sin(7x) \qquad \text{(IC)}$$

for all $0 < x < \pi$ and $0 < t < 1$.

(a) Suppose that $u(x,t) = \sum_{n=1}^{\infty} b_n(t)\sin(nx)$ satisfies (PDE). Find, for each $1 \le n < \infty$, an ordinary differential equation satisfied by $b_n(t)$.

(b) Find, for each $1 \le n < \infty$, the general solution to the ODE of part (a).

(c) Find $u(x,t)$ for all $0 < t < 1$ and $0 < x < \pi$.

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[15] **3.** Let $a[n]$ and $b[n]$ be the periodic discrete–time signals with period $N = 6$ determined by

$$a[0]=1 \quad a[1]=0 \quad a[2]=1 \quad a[3]=0 \quad a[4]=1 \quad a[5]=0$$
$$b[0]=0 \quad b[1]=1 \quad b[2]=0 \quad b[3]=1 \quad b[4]=0 \quad b[5]=1$$

Let $c[n] = (a * b)[n]$ be their convolution. Compute

$$\widehat{a}[k], \qquad \widehat{b}[k], \qquad c[n] \qquad \text{and} \qquad \widehat{c}[k]$$

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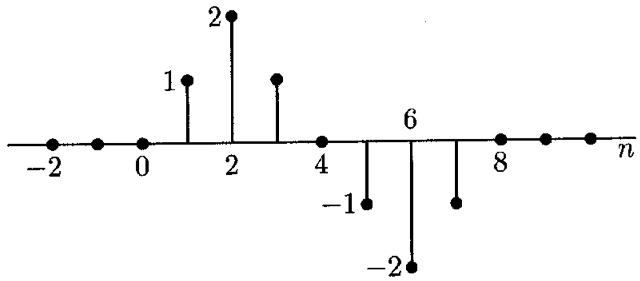
[15] **4.** All nonzero values of a discrete–time signal $h[n]$ are shown in the sketch below.



(a) Find $\widehat{h}(0)$.

(b) Prove that $|\widehat{h}(\omega)| = 2\big|\sin\omega + 2\sin(2\omega) + \sin(3\omega)\big|$.

(c) Find $\widehat{u}(\omega) = \mathrm{Re}\left(\widehat{h}(\omega)\right)$, assuming that ω is real.

(d) Suppose that $h[n]$ is the impulse response function of a certain system. **Find and sketch** the output $y[n]$ of the system when the input is

$$x[n] = \begin{cases} \frac{1}{2} & \text{if } n = 0, \pm 3, \pm 6, \pm 9, \cdots \\ 0 & \text{otherwise} \end{cases}$$

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[15] **5.** Consider a discrete–time, linear, time-invariant (LTI) system whose impulse response function $h[n]$ has z–transform $H(z)$.

(a) Define in words what it means to say that such a system is causal. What special property must $h[n]$ have for an LTI system to be causal?

(b) Define what it means to say that such a system is stable. What special property must $h[n]$ have for an LTI system to be stable?

(c) Find $H(z)$ if $h[n] = \left(\frac{3}{2}\right)^n (u[n+2] - u[n-2])$.

(d) Find $h[n]$ if $H(z) = \dfrac{5z-4}{z^2-z-2}$ and $h[n]$ is causal.

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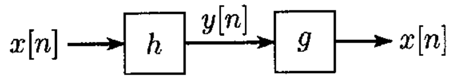
[10] **6.** The input and output signals, $x[n]$ and $y[n]$ respectively, of a causal LTI system are related by

$$y[n] = ay[n-2] + bx[n]$$

where a and b are constants.

(a) Find the impulse response function, $h[n]$, for this LTI system.

(b) A second causal LTI system, with impulse response function $g[n]$, is to be the inverse of the system of part (a). This means that if the two systems are cascaded so that the output of the h system is fed into the input of the g system as in the figure below,



then the output of the g system is to be the original input of the h system. Find $g[n]$.

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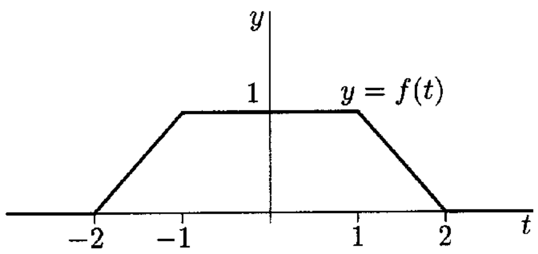
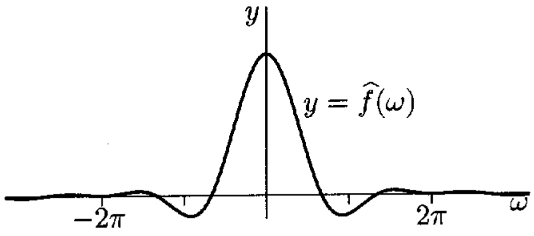
[15] **7.** The sketches below show a signal $f(t)$ and its Fourier transform $\widehat{f}(\omega)$.





(a) Find $\widehat{f}(0)$.

(b) Give a formula for $\widehat{f}(\omega)$, $\omega \neq 0$. (Simplify your formula until i does not appear.)

(c) Let $g(t) = f(t)\sin(6\pi t)$. Find $\widehat{g}(0)$ and sketch the graphs of g and $\widehat{g}$. Label the axes with care.

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