

The University of British Columbia

Final Examination - December 2007

Mathematics 265

Section 103

Closed book examination

Time: 2.5 hours

Last Name: _____ First: _____ Signature _____

Student Number _____

Special Instructions:

- Be sure that this examination has 11 pages. Write your name on top of each page.
- You are allowed to bring into the exam one $8\frac{1}{2} \times 11$ formula sheet filled on both sides. No calculators or any other aids are allowed.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
 - (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
 - (b) Speaking or communicating with other candidates.
 - (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		10
2		10
3		20
4		15
5		15
6		15
7		15
Total		100

[10] 1. Find all solutions of $y' - 2xy^2 = 0$.

[10] **2.** Solve the initial value problem $xy' = x^3 - 2y$, $y(1) = 0$.

[20] **3.** Consider the initial value problem

$$y'' + ay' + by = 0, \quad y(0) = 3, \quad y'(0) = 5.$$

The differential equation has as a fundamental set of solutions $\{y_1(t), y_2(t)\}$, where $y_1(t) = e^{-t}$. The Wronskian of y_1 and y_2 is $W(t) = 4e^{2t}$.

- (a) Solve for $y_2(t)$.
- (b) Determine the values of the constants a and b .
- (c) Solve the initial value problem.

Extra space (if needed)

[15] 4. The homogeneous differential equation

$$t^2 y'' - 2ty' + 2y = 0,$$

defined over the open interval $0.5 < t < 2$, has a non-trivial solution $y_1 = t^2$.

- (a) Use reduction of order to find a second solution y_2 .
- (b) Show that y_1 and y_2 form a fundamental set of solutions.
- (c) Find the particular solution that satisfies the initial conditions $y(1) = 3$ and $y'(1) = 4$.

Extra space (if needed)

[15] 5. Solve the initial value problem

$$y'' + 2y' + 5y = f(t), \quad y(0) = 1, \quad y'(0) = -1,$$

where

$$f(t) = \begin{cases} 0 & \text{if } t < 1; \\ 1 & \text{if } 1 \leq t. \end{cases}$$

Extra space (if needed)

[15] **6.** Solve the initial value problem

$$\begin{aligned}x_1' &= x_1 - x_2 \\x_2' &= 5x_1 - 3x_2\end{aligned}$$

with $x_1(0) = 1$, $x_2(0) = 3$. Describe the behaviour of the solution as $t \rightarrow \infty$.

[15] 7. Find a fundamental matrix for the system of equations

$$\mathbf{x}' = \begin{pmatrix} 1 & -2 \\ 2 & 5 \end{pmatrix} \mathbf{x}.$$