Be sure that this examination has 5 Questions and 17 pages including this cover

The University of British Columbia

Final Examinations - 14 December, 2009

Mathematics 257/316

All Sections

Time: 2.5 hours

First Name (USE CAPITALS)	____________	**Last Name** (USE CAPITALS)	____________
Signature	____________	**Instructor's Name**	____________
Student Number	____________	**Section Number**	____________

Special Instructions:

Students are not allowed to bring any notes into the exam.
No calculators are allowed.

1		20
2		25
3		15
4		20
5		20
Total		100

Rules governing examinations:

1. **Each candidate should be prepared to produce his/her library/AMS card upon request.**

2. **Read and observe the following rules:**
   No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.
   Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
   CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
   (a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
   (b) Speaking or communicating with other candidates.
   (c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. **Smoking is not permitted during examinations.**

1. Consider the differential equation

$$6x^2y'' + 7xy' - (1+x)y = 0 \tag{1}$$

(a) Classify the points $0 \leq x < \infty$ as ordinary points, regular singular points, or irregular singular points.

(b) Find two values of r such that there are solutions of the form $y(x) = \sum_{n=0}^{\infty} a_n x^{n+r}$.

(c) Use the series expansion in (b) to determine two independent solutions of (1). You only need to calculate the first three non-zero terms in each case.

[20 marks]

(Question 1 Continued)

(Question 1 Continued)

2. Consider the following initial boundary value problem for the heat equation:

$$\begin{aligned} u_t &= u_{xx} - 4u \,, \qquad 0 < x < \pi \,, \qquad t > 0 \\ u_x(0,t) &= 0 \,, \qquad u_x(\pi,t) = 1 \\ u(x,0) &= 4\cos(4x) \end{aligned} \tag{2}$$

(a) Determine a steady state solution to the boundary value problem. [5 marks]

(b) Use this steady state solution to determine the solution to the boundary value problem (2) by separation of variables. [12 marks]

(c) Briefly describe how you would use the method of finite differences to obtain an approximate solution this boundary value problem. Use the notation $u_n^k \simeq u(x_n, t_k)$ to represent the nodal values on the finite difference mesh. Explain how you propose to approximate the boundary condition $u_x(\pi,t) = 1$. [8 marks]

Hint: It might be useful to know that $\int_0^\pi \cosh(2x)\cos(nx)\,dx = \frac{2(-1)^n \sinh(2\pi)}{4+n^2}$

[total 25 marks]

(Question 2 Continued)

(Question 2 Continued)

3. Consider the following initial-boundary value problem:

$$\begin{aligned} u_{tt} + k^2 u &= c^2 u_{xx},\ 0 < x < \pi \\ u(0,t) &= u(\pi,t) = 0 \\ u(x,0) &= f(x),\ u_t(x,0) = 0. \end{aligned}$$

(a) Use separation of variables to determine the solution to this boundary value problem.

(b) If $k = 0$, $c = \pi$, and if the initial displacement $f(x) = \sin(x)$, determine the shape of the string at time $t = 1/2$. [15 marks]

(Question 3 Continued)

(Question 3 Continued)

4. Use separation of variables to solve the following mixed boundary value problem for a wedge shaped region in the first quadrant:

$$\begin{aligned} u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} &= 0, \quad 0 < r < a, \quad 0 < \theta < \pi/2 \\ u(r,0) &= 0 \quad \text{and} \quad \frac{\partial u}{\partial \theta}(r, \pi/2) = 1 \\ u(a,\theta) &= 1 + \theta \quad \text{and} \quad u(r,\theta) \overset{r\to 0}{<} \infty \end{aligned}$$

[20 marks]

(Question 4 Continued)

(Question 4 Continued)

5. Solve the inhomogeneous heat conduction problem:

$$\begin{aligned} u_t &= u_{xx} + \cos(t)\cos(x),\ 0 < x < \pi/2,\ t > 0 \\ u_x(0,t) &= u(\pi/2,t) \ = \ 0 \\ u(x,0) &= 0. \end{aligned}$$

Hint: It may be useful to know that

$$\int e^{\gamma t} \cos t \, dt = \frac{e^{\gamma t}(\gamma \cos(t) + \sin(t))}{\gamma^2 + 1}$$

[20 marks]

(Question 5 Continued)

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