Be sure this exam has 10 pages including the cover

The University of British Columbia

MATH 256, Sections 102 and 103

Final Exam – December 4, 2014

Name ______________________________ **Signature** ______________________________

Student Number ____________________

This exam consists of **8** questions. No notes. Simple numerics calculators are allowed. List of Laplace Transform is provided. Write your answer in the blank page provided.

Problem	max score	score
1.	15	
2.	10	
3.	10	
4.	10	
5.	10	
6.	20	
7.	10	
8.	15	
total	100	

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

(15 points) 1. Solve the following ordinary differential equation

$$\frac{dy}{dx} = \frac{y^2 + 2xy}{x^2}, \qquad y(1) = 1$$

(10 points) 2. Find the critical points of the following population model

$$\frac{dy}{dx} = \left(y^2 - 1\right)\left(e^y - 1\right)$$

and classify the stability/instability of these critical points.

(10 points) 3. Use whatever method to solve the following second order differential equation

$$y^{''} - 3y^{'} + 2y = e^{t}, \quad y(0) = 1, y^{'}(0) = 0$$

(10 points) 4. Consider the following second order equation

$$ty'' - y' + 4t^3 y = 0, \quad t > 0$$

Check that $y_1 = \cos(t^2)$ is a solution. Use the reduction of order to find the second solution of the homogeneous problem.

Hint: You may use the integration formula $\int \frac{1}{cos^2(x)} dx = \tan(x)$.

(15 points) 5. Use whatever method to obtain the general solutions of

$$\mathbf{x}' = \begin{pmatrix} 1 & -3 \\ -2 & 2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} t-2 \\ 2t \end{pmatrix}$$

(20 points) 6. Use the method of Laplace transform to solve

$$y'' + 2y' + 2y = 2u_1(t) + e^t\delta(t-2), \quad y(0) = 1, y'(0) = 0$$

(10 points) 7. Consider the following function

$$f(x) = \begin{cases} 1, & -1 \leq t < 0 \\ x, & 0 \leq x < 1 \end{cases} \qquad f(x+2) = f(x)$$

(7 points) (a) Compute the first three coefficients of full Fourier series expansion a_0, a_1, b_1.

(3 points) (b) Find out the values of the full Fourier series expansion at $x = -\frac{1}{2},\ 0,\ \frac{1}{2}$.

(15 points) 8. Use the method of separation of variables to solve the following heat equation

$$u_t = u_{xx}, \quad 0 < x < \pi, \ t > 0;$$

$$u(x, 0) = 2\cos^2(x), \quad 0 < x < \pi;$$

$$u_x(0, t) = 0, \quad u_x(\pi, t) = 0, \ t > 0.$$