Marks

[20] **1.** Linear differential equations.

    (a) Solve the initial value problem:

$$y' + 3y = 2e^{-2t}, \quad y(0) = 1.$$

    (b) Solve the initial value problem:

$$y' + 2ty = e^{-t^2}, \quad y(0) = 3.$$

(c) Solve the initial value problem:

$$y'' + 2y' + y = 0, \quad y(0) = 0, \ y'(0) = 2.$$

(d) Solve the initial value problem:

$$y'' + 2y' + y = 2e^{-t}, \quad y(0) = 0, \ y'(0) = 0,$$

and show that the solution $y(t)$ has maximum value of $4e^{-2}$. At what time is this value attained?

Marks

[20] **2.** Find the general solution of the following homogeneous linear system:

$$\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 5 & 3 \end{pmatrix} \mathbf{x},$$

and sketch the phase plane close to $\mathbf{x} = 0$.

(b) Find the solution of the following initial value problem:

$$\mathbf{x}' = \begin{pmatrix} 1 & 3 \\ 5 & 3 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 12t - 11 \\ -3 \end{pmatrix}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

[20] **3.** The function $f(t)$ is defined for $t \in [-1, 1]$ by

$$f(t) = \begin{cases} -t - 1, , & t \in [0, 1], \\ 1 - t, & t \in [0, 1], \end{cases}$$

(a) Sketch the function $f(t)$ over the interval $t \in [-1, 1]$, and find the Fourier series for $f(t)$. Is the function odd or even?

(b) To what values should the Fourier series of part (a) converge to at: (i) $t = 0.5$; (ii) $t = 2$; (iii) $t = 3$; (iv) $t = 3.5$?

(c) Find the general solution of:
$$y'' + \omega^2 y = f(t),$$

where $f(t)$ is the function in part (a), assuming ω is a positive constant satisfying $\omega < \pi$.

(d) If instead we have $\omega \geq \pi$, for what value(s) of ω will your solution become very large as $t \to \infty$?

[20] **4.** Consider the the following IBVP

$$u_t = u_{xx} - \cos 3\pi x, \quad 0 < x < 1, \ t \geq 0,$$

subject to the boundary and initial conditions:

$$u_x(0,t) = 2, \quad u_x(1,t) = 2, \quad u(x,0) = 2x + \cos \pi x$$

(a) Writing $u(x,t) = u_s(x) + v(x,t)$ state the problems that are satisfied by the steady state solution, $u_s(x)$, and the transient solution $v(x,t)$.

(b) Find the steady state solution, $u_s(x)$.

(c) Find the transient solution, $v(x,t)$ using separation of variables, and hence write down the full solution, $u(x,t)$.

(d) Show that

$$u(0.25, t) \to \frac{1}{2} + \frac{1}{\sqrt{2}}, \text{ as t} \to \infty,$$

and estimate how large t must be in order for

$$\left| u(0.25, t) - \frac{1}{2} - \frac{1}{\sqrt{2}} \right| < 0.01.$$

[20]   **5.**   Use separation of variables to solve the BVP:

$$u_{xx} + u_{yy} = 0, \quad 0 < x < 1, \ 0 < y < 1,$$

subject to the boundary conditions: $u(0, y) = 1$, $u_x(1, y) = 0$, $u(x, 0) = 0$, $u(x, 1) = \sin 5\pi x$.

The End

Be sure that this examination has 11 pages including this cover

The University of British Columbia

Sessional Examinations - April 2009

Mathematics 256

Differential Equations

Closed book examination   Time: $2\frac{1}{2}$ hours

Name _____   **Signature** _____

Student Number _____   **Instructor's Name** _____

Section Number _____

Special Instructions:

Two $8\frac{1}{2}$" × 11" two sided cheat sheets are permitted.
Non-programmable calculators allowed.
Show your work in the spaces provided.
You are encouraged to explain all steps in your solutions.

Rules Governing Formal Examinations

1		20
2		20
3		20
4		20
5		20
Total		100