Math 253 Final Exam

Dec 9, 2009

Duration: 150 minutes

Last Name: ____________ First Name: ____________ Student Number: ________

Do not open this test until instructed to do so! This exam should have 14 pages, including this cover sheet. It is a closed book exam; no textbooks, calculators, laptops, formula sheets or other aids are allowed. Turn off any cell phones, pagers, etc. that could make noise during the exam. You must remain in this room until you have finished the exam. **Circle your solutions! Reduce your answer as much as possible. Explain your work. Relax.** Use the extra pages if necessary.

Read these UBC rules governing examinations:

(i) Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.

(ii) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

(iii) No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

(iv) Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

- Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
- Speaking or communicating with other candidates.
- Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

(v) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Problem	**Out of**	**Score**
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
Total	100	

Problem 1 (5+5 points)

Consider the three points $A = (-1, 0, 2)$, $B = (4, 1, 3)$ and $C = (0, 2, -1)$.

a) Find the equation of the plane containing A, B and C.

b) Find the area of the triangle with vertices A, B and C.

Problem 2 (10 points)

Find the points on the circle $x^2 + y^2 = 80$ which are closest to and farthest from the point $P = (1, 2)$. *Hint: You may use the method of Lagrange multipliers.*

Problem 3 (5+5 points)

a) Let the functions $f(x, y)$ and $g(x, y)$ be such that $f_x = g_y$ and $f_y = -g_x$. Show that $f_{xx} + f_{yy} = 0$ and $g_{xx} + g_{yy} = 0$.

b) Do the functions $f(x, y) = \exp(x)\cos(y)$ and $g(x, y) = \exp(-x)\sin(y)$ satisfy the conditions in a)?

Problem 4 (4+6 points)

Consider the function $f(x, y) = x^3 + y^3 - 3xy$.

a) Find all critical points of f.

b) Find all local minima, maxima and saddle points of f.

Problem 5 (2+2+6 points)

Let R be the region in the xy-plane bounded by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > 0,\ b > 0.$$

Consider the coordinate transformation $x = ar\cos(\theta)$ and $y = br\sin(\theta)$.

a) Describe R in the new coordinates (r, θ).

b) Compute the Jacobian $\frac{\partial(x,y)}{\partial(r,\theta)}$.

c) Compute the area of R using the coordinates (r, θ).

Problem 6 (10 points)

Use spherical coordinates to compute the triple integral

$$\iiint_R \frac{1}{\sqrt{x^2+y^2+z^2}}\, dV$$

where $R = \{\, (x, y, z) \;:\; 1 \leq x^2 + y^2 + z^2 \leq 4 \,\}$.
Hint: In spherical coordinates, we have $dV = \rho^2 \sin(\phi)\, d\rho\, d\theta\, d\phi$.

Problem 7 (5+5 points)

Consider the ellipsoid $x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 3$.

a) Find the equations of the tangent planes at $P_1 = (1, -2, -3)$ and $P_2 = (1, 2, 3)$.

b) Find the parametric equations of the line of intersection of the two planes.

Problem 8 (5+5 points)

a) Find the limit: $\lim_{(x,y)\to(0,0)} \dfrac{x^8 + y^8}{x^4 + y^4}$.

b) Prove that the following limit does not exist: $\lim_{(x,y)\to(0,0)} \dfrac{xy^5}{x^8 + y^{10}}$.

Problem 9 (5+5 points)

Consider the integral

$$\iint_B \frac{dA}{(x^2+y^2)^k}$$

with $B = \{\,(x,y)\,|\,x^2+y^2 \leq 1\,\}$.

a) For what values of k does the integral exist?

b) Evaluate the integral for these values.

Problem 10 (4+3+3 points)

a) Draw $xy = 1$ in Cartesian coordinates (x, y, z).

b) Draw $\phi = 3\pi/4$ in spherical coordinates (ρ, θ, ϕ).

c) Draw $r = \theta$ in cylindrical coordinates (r, θ, z).

Extra page

Extra page

Extra page