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The University of British Columbia
Sessional Examinations - April 2016

Mathematics 227
Advanced Calculus II

Closed book examination

Time: $2\frac{1}{2}$ hours

Name ______________________ **Signature** ______________________

Student Number ______________________ **Instructor's Name** ______________________

Section Number ______________________

Special Instructions:

No books, notes, or calculators are allowed.

Rules Governing Formal Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.

2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.

3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.

5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:

i. speaking or communicating with other examination candidates, unless otherwise authorized;

ii. purposely exposing written papers to the view of other examination candidates or imaging devices;

iii. purposely viewing the written papers of other examination candidates;

iv. using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,

v. using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s) — (electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).

6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.

7. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

1		12
2		12
3		12
4		12
5		12
6		12
7		12
8		16
Total		100

Marks

[12] **1.** A baseball pitcher throws a curve ball. As it leaves his hand, the ball is travelling with velocity $\vec{\mathbf{v}}(0) = \vec{\mathbf{v}}_0 \neq 0$ and (vector) angular velocity $\vec{\Omega} \neq 0$ with $\vec{\mathbf{v}}_0$ perpendicular to $\vec{\Omega}$. The angular velocity remains constant throughout the flight. The acceleration of the ball is

$$\vec{\mathbf{a}}(t) = 3\vec{\Omega} \times \vec{\mathbf{v}}(t)$$

where $\vec{\mathbf{v}}(t)$ is the velocity vector at time t.

(a) Show that $|\vec{\mathbf{v}}(t)|$ is constant in t.

(b) Show that $\vec{\Omega} \cdot \vec{\mathbf{v}}$ is constant in time.

(c) Find the curvature κ. Hint: Use $\vec{\mathbf{a}} = \frac{d^2s}{dt^2}\hat{\mathbf{T}} + \kappa|\vec{\mathbf{v}}|^2\hat{\mathbf{N}}$.

(d) Find the Frenet frame $\hat{\mathbf{T}}, \hat{\mathbf{N}}, \hat{\mathbf{B}}$ in terms of $\vec{\Omega}$ and $\vec{\mathbf{v}}(t)$.

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[12] **2.** Consider the vector field $\vec{\mathbf{V}} = \frac{2y}{x}\hat{\boldsymbol{\imath}} - \hat{\boldsymbol{\jmath}}$ on the domain $\{\ (x, y) \in \mathbb{R}^2 \mid x > 0\ \}$.

(a) Sketch the stream lines for this vector field.

(b) At time $t = 0$, a piece of wood is placed in the stream at $(3, 0)$. At what time, if ever, does it reach $(1, -2)$?

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[12] **3.** Let $\vec{\mathbf{F}} = e^x \sin y\, \hat{\imath} + [ae^x \cos y + bz]\, \hat{\jmath} + cx\, \hat{\mathbf{k}}$. Let P be the path which starts at $(0,0,0)$, ends at $(1,1,1)$ and follows

$$x - 2y + z = 0$$
$$3x - y - 2z = 0$$

(a) For which values of the constants $a,\ b,\ c$ is $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = 0$ for all closed paths C?

(b) Evaluate $\int_P \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ when $a = 1,\ b = 0,\ c = 0$.

(c) Evaluate $\int_P \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ when $a = 1,\ b = 1,\ c = 1$.

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[12] **4.** Let S be the surface

$$x^2 + y^2 = e^{2z} \qquad 0 \le z \le 1 \quad y > 0$$

(a) Find the flux of $\vec{\mathbf{F}} = yz\,\hat{\boldsymbol{\imath}} - xz\,\hat{\boldsymbol{\jmath}} + (x^2 + y^2)\hat{\mathbf{k}}$ through S using the normal to S having positive vertical component.

(b) Find the total charge on S given that the charge density is $\rho = \sqrt{1 + e^{2z}}$.

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[12] **5.** The sides of a grain silo are given by the portion of the cylinder $x^2 + y^2 = 1$ where $0 \le z \le 1$. The top of the silo is given by the portion of the sphere $x^2 + y^2 + z^2 = 2$ lying within the cylinder and above the xy–plane. Find the flux of the vector field

$$\vec{\mathbf{F}}(x, y, z) = (x^2yz\,,\, e^x z\,,\, x^2 + y)$$

out of the silo.

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[12] **6.** Evaluate the integral $\oint_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, in which $\vec{\mathbf{F}} = (e^{x^2} - yz, \sin y - yz, xz + 2y)$ and C is the triangular path from $(1, 0, 0)$ to $(0, 1, 0)$ to $(0, 0, 1)$ and back to $(1, 0, 0)$.

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[12] **7.** Set

$$M(x,y) = \frac{2y}{(x+1)^2+y^2} - \frac{5y}{(x-1)^2+y^2} \qquad N(x,y) = -\frac{2(x+1)}{(x+1)^2+y^2} + \frac{5(x-1)}{(x-1)^2+y^2}$$

Evaluate $\int_C \big(M\,dx + N\,dy\big)$ where C is the triangular path from $(0,0)$ to $(2,1)$ to $(2,-1)$ and back to $(0,0)$.

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[16] **8.** The following statements may be true or false. Decide which. If true, give a proof. If false, provide a counter–example.

(a) If $\vec{\mathbf{F}}$ is any smooth vector field defined in $\mathbb{R}^3$ and if S is any sphere, then $\iint_S \vec{\nabla}\times\vec{\mathbf{F}}\cdot\hat{\mathbf{n}}\,dS = 0$. Here $\hat{\mathbf{n}}$ is the outward normal to S.

(b) If $\vec{\mathbf{F}}$ and $\vec{\mathbf{G}}$ are smooth vector fields in $\mathbb{R}^3$ and if $\oint_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \oint_C \vec{\mathbf{G}}\cdot d\vec{\mathbf{r}}$ for every circle C, then $\vec{\mathbf{F}} = \vec{\mathbf{G}}$.

(c) Let $\vec{\mathbf{F}}$ and $\vec{\mathbf{G}}$ be smooth vector fields defined in $\mathbb{R}^3$. Suppose that, for every circle C, we have $\oint_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \iint_S \vec{\mathbf{G}}\cdot\hat{\mathbf{n}}\,dS$, where S is the oriented disk with boundary C. Then $\vec{\mathbf{G}} = \vec{\nabla}\times\vec{\mathbf{F}}$.

(d) If $\vec{\mathbf{F}}$ is a conservative vector field and ϕ a scalar function, then the vectors $\vec{\mathbf{F}}$ and $\vec{\nabla}\times(\phi\vec{\mathbf{F}})$ are perpendicular.

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