**Be sure that this examination has 10 pages including this cover**

**The University of British Columbia**

Sessional Examinations - December 2011

**Mathematics 226**

*Advanced Calculus I*

Closed book examination Time: $2\frac{1}{2}$ hours

**Name** _____ **Signature** _____

**Student Number** _____ **Instructor's Name** _____

**Section Number** _____

## Special Instructions:

No books, notes, or calculators are allowed.

### Rules Governing Formal Examinations

**1.** Each candidate must be prepared to produce, upon request, a UBCcard for identification.

**2.** Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

**3.** No candidate shall be permitted to enter the examination room after the expiration of one half hour from the scheduled starting time, or to leave during the first half hour of the examination.

**4.** Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

(a) having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;

(b) speaking or communicating with other candidates; and

(c) purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.

**5.** Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

**6.** Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

| | | |
|---|---|---|
| 1 | | 12 |
| 2 | | 12 |
| 3 | | 15 |
| 4 | | 12 |
| 5 | | 12 |
| 6 | | 15 |
| 7 | | 12 |
| 8 | | 10 |
| Total | | 100 |

Marks

[12] **1.** Consider the ellipsoid

$$x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 21$$

(a) Find an equation for the tangent plane at the point $(a, b, c)$ on the ellpsoid.

(b) At which points on the ellipsoid is the tangent plane parallel to the plane $x + 4y + \frac{4}{3}z = 471$?

(c) For which points on the ellipsoid does the tangent plane pass through $(7, 0, 0)$? Describe this set geometrically.

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[12] **2.** A gas is known to satisfy the van der Waals law $p = \frac{2T}{V-1} - \frac{4}{V^2}$ where $p$ =pressure, $V$ =volume and $T$ =temperature. This equation of state is used to determine $V$ as a function of $p$ and $T$ near $p = T = 1$. In particular $V(1,1) = 2$.

(a) Determine $\frac{\partial V}{\partial p}(1,1)$, $\frac{\partial V}{\partial T}(1,1)$ and $\frac{\partial^2 V}{\partial p^2}(1,1)$.

(b) Measurements are made and it is found that $p = 1 \pm 0.02$ and $T = 1 \pm 0.02$. Find the approximate maximum error in the value of $V$ if we take $V = 2$.

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[15] **3.** Consider the function

$$f(x, y) = 2x^3 - 6xy + y^2 + 4y$$

(a) Find and classify all of the critical points of $f(x, y)$.

(b) Find the maximum and minimum values of $f(x, y)$ in the triangle with vertices $(1, 0)$, $(0, 1)$ and $(1, 1)$.

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[12] **4.** Find the points on the ellipse $2x^2 + 4xy + 5y^2 = 30$ which are closest to and farthest from the origin.

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[12] **5.** Reverse the order of integration of

$$\int_0^1 \left( \int_0^y \frac{e^x}{1-x} dx \right) dy + \int_1^2 \left( \int_0^{2-y} \frac{e^x}{1-x} dx \right) dy$$

and then evaluate the integral.

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[15] **6.** Consider the integral

$$\int_0^1 dy \int_y^{1+\sqrt{1-y^2}} dx\ \sqrt{x}$$

(a) Sketch the region of integration.

(b) Express the integral in polar coordinates.

(c) Evaluate it.

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[12] **7.** A cylindrical hole of radius $a$ cm is drilled through the centre of a solid sphere of radius $b$ cm (where $b > a$) and density $\rho\, y^2$ gm/cm$^3$. Find the mass of the *remaining part* of the sphere.

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[10] **8.** Consider the function

$$f(x,y) = \begin{cases} \frac{x^3}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) Evaluate, if possible, $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$.

(b) Is $f(x,y)$ differentiable at $(0,0)$? You must justify your answer to receive any marks.

**The End**