Marks

[12] **1.**

(a) (4 marks) Sketch the hyperboloid $z^2 = 4x^2 + y^2 - 1$.

(b) (8 marks) Find all points on the hyperboloid $z^2 = 4x^2 + y^2 - 1$ where the tangent plane is parallel to the plane $2x - y + z = 0$.

Continued on page 3

[8] **2.** A bug walks on a flat horizontal metal plate where the temperature is given by a C^1 function $T(x, y)$. At a certain time, she is at the point $(1, 0)$ in the plane (all distances are measured in meters). If she walks north (in the direction of $(0, 1)$) from that point, the temperature will increase at a rate of 3 degrees per meter. If she walks southeast (in the direction of $(1, -1)$), the temperature will remain constant. In what direction should the bug walk so as to cool off as quickly as possible, and what will be the rate of change of temperature (in degrees per meter) in that direction?

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[16] **3.** Let $f(x, y) = e^{-x}(2x - y^2)$.

(a) (6 marks) This function has exactly one critical point. Find it.

(b) (6 marks) Find the second order Taylor polynomial of $f(x, y)$ at the critical point found in (a).

(c) (4 marks) Does $f(x, y)$ have a minimum, maximum, or a saddle point at this critical point?

Continued on page 5

[12] **4.** The plane $x - y + 2z = 6$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are closest to and farthest from the origin.

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[12] **5.** Evaluate the following integrals:

(a) (6 marks) $\int\int_D 2x dA$, where D is the triangle in the xy-plane with vertices $(0,0)$, $(1,0)$, $(3,1)$;

(b) (6 marks) $\int\int_D y dA$, where D is the region in the xy-plane given by $D = \{(x,y) : \ 0 \leq y \leq x, \ x^2 + y^2 \leq 9\}$. (Hint: use polar coordinates.)

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[16] **6.** The integral $\int_0^1 \int_0^{(y-1)^2} \int_0^{2-2z} 1 \, dx \, dz \, dy$ represents the volume of a three-dimensional region.

(a) (4 marks) Sketch the region of integration.

(b) (6 marks) Change the order of integration to get an integral of the form $\int \int \int dy \, dx \, dz$.

(c) (6 marks) Evaluate the integral in (b).

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[12] **7.** Evaluate each limit or prove that it does not exist.

(a) (6 marks) $\lim_{(x,y)\to(0,0)} \dfrac{x^3 - y^3}{x^2 + y^2}$

(b) (6 marks) $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - y^4}{x^2 + y^4}$

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[12] **8.** Let

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

(a) (4 marks) Find $\dfrac{\partial f}{\partial x}(0,0)$ and $\dfrac{\partial f}{\partial y}(0,0)$.

(b) (8 marks) Prove that f is not differentiable at $(0,0)$.

The End

Be sure that this examination has 10 pages including this cover

The University of British Columbia

Sessional Examinations - December 2009

Mathematics 226

Advanced Calculus I

Closed book examination Time: 2.5 hours

Print Name _____ **Signature** _____

Student Number _____ **Instructor's Name** _____

Section Number _____

Special Instructions:

No calculators, notes, or books of any kind are allowed.
Show all calculations for your solutions. If you need more space than is provided, use the back of the previous page.

Rules governing examinations

1. Each candidate should be prepared to produce his library/AMS card upon request.
2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

1		12
2		8
3		16
4		12
5		12
6		16
7		12
8		12
Total		100