1. (8 points) Compute the following limits or explain why they do not exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$.

(b) $\displaystyle\lim_{(x,y)\to(0,0)} |y|^x$.

(c) $\displaystyle\lim_{(x,y)\to(-1,1)} \frac{x^2+2xy^2+y^4}{1+y^4}$.

(d) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{\sin xy}{x^2+y^2}$.

2. (12 points) Suppose that a planet moves around the sun in a circular orbit of radius $r > 0$ with the sun at the center. By Kepler's third law, the period T of the orbit (i.e., the length of a year on the planet) is given by

$$T^2 = \alpha r^3$$

where α is a positive constant.

(a) Using Kepler's second law, show that the speed of the planet is constant. (Hint: As explained in class, Kepler's second law states that the orbit sweeps out equal area in equal times.)

(b) Show that the acceleration, $\vec{a} = \ddot{\vec{r}}$, of the planet is given by

$$\vec{a} = C\frac{\vec{r}}{r^3}$$

for some constant C depending on α. Determine C in terms of α.

3. (15 points) Let $f(x,y) = xy(5x+y-15)$.

(a) Find all critical points and classify them as local minima, local maxima or saddle points.

(b) Does f have any global minima or maxima on $\mathbb{R}^2$. If it does, compute them.

(c) Does f have any global minima or maxima on $\{(x,y) \in \mathbb{R}^2 \,|\, x \geq 0, y \geq 0\}$. If it does, compute them.

4. (10 points) Let $z = f(x,y)$ and set $x = 3s + 2t, y = s + 2t$. Find the values of the constants a, b and c such that

$$a\frac{\partial^2 z}{\partial x^2} + b\frac{\partial^2 z}{\partial x \partial y} + c\frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial s^2} + \frac{\partial^2 z}{\partial t^2}.$$

5. (15 points) Let $(a_1, \ldots, a_n) \in \mathbb{R}^n$ and let $f : \mathbb{R}^n \to \mathbb{R}$ be the linear function given by

$$f(x_1, \ldots, x_n) = \sum_{i=1}^{n} a_i x_i.$$

(a) Compute the minimum and maximum values of f on the ball of radius r centered at the origin in $\mathbb{R}^n$.

(b) Now compute the minimum and maximum values of f on the ball of radius r centered at a point $\vec{y} = (y_1, \ldots, y_n) \in \mathbb{R}^n$.

(c) Now let $g : \mathbb{R}^3 \to \mathbb{R}$ be the function given by $g(x,y,z) = 5x + 3y + 2z$. Compute the minimum and maximum values of g on the ball of radius 5 centered at the point $(1,1,1)$.

6. (20 points) Let

$$f(x,y) = \begin{cases} \frac{x^2y}{x^2+y^2} & (x,y) \neq (0,0); \\ 0 & \text{else.} \end{cases}$$

(a) Use the definition of partial derivatives to compute $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at $(0,0)$.

(b) Let a be a non-zero constant and let $\vec{x}(t) = (t, at)$. Show that $f \circ \vec{x} : \mathbb{R} \to \mathbb{R}$ is differentiable and compute $D(f \circ \vec{x})(0)$.

(c) Now compute $Df(0,0) \circ D\vec{x}(0)$.

(d) Is f differentiable at $(0,0)$?

7. (20 points) Suppose $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a function.

(a) State the definition of

$$\lim_{(x,y)\to(0,0)} f(x,y).$$

(b) State the definition of the derivative Df of f at $(0,0)$.

Math 226 | Final Exam | Fall 2007

Patrick Brosnan, Instructor

First Name/Last Name:______________________________

Student ID Number:______________________________

Section/Professor:______________________________

Signature:

By signing here, you confirm you are the person identified above and that all the work herein is solely your own.

Instructions:

(1) No calculators, books, notes, or other aids allowed.
(2) Give your answer in the space provided. If you need extra space, use the back of the page. **PLEASE BOX ALL FINAL ANSWERS!** And **clearly indicate whether you are planning to prove a statement or give a counterexample at the beginning of the problem.**
(3) Show enough of your work to justify your answer. Show ALL steps.

Problem	Points	Score
1	8	
2	12	
3	15	
4	10	
5	15	
6	20	
7	20	
Total	100	