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The University of British Columbia
Sessional Examinations - December 2000

Mathematics 226
Advanced Calculus I

Closed book examination

Time: $2\frac{1}{2}$ hours

Name _____ Signature _____

Student Number _____ Instructor's Name _____

Section Number _____

Special Instructions:

Calculators are permitted.

Rules Governing Formal Examinations

1. Each candidate must be prepared to produce, upon request, a UBCcard for identification.
2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
3. No candidate shall be permitted to enter the examination room after the expiration of one half hour from the scheduled starting time, or to leave during the first half hour of the examination.
4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:
 - (a) having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
 - (b) speaking or communicating with other candidates; and
 - (c) purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		10
2		15
3		15
4		15
5		10
6		10
7		10
8		15
Total		100

Marks

- [10] 1. Let S be the surface $x^2 + 4y^2 - 9z^2 = 1$.
- Sketch the surface.
 - Find the equation of the tangent plane to the surface at $(1, 3, 2)$.
 - Suppose that a smooth curve C lies on the surface and passes through the point $(1, 3, 2)$. Suppose also that the tangent line L to the curve C at $(1, 3, 2)$ intersects the x -axis at some point Q . Determine the coordinates of Q .

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- [15] 2. The height of land in the vicinity of a hill is given, in terms of horizontal coordinates (x, y) , by

$$h(x, y) = \frac{12000}{x^2 + 2x + 4y^2 + 11}$$

A parking lot is to be built at $(4, 5)$ and a hiking trail is to be made. The trail is to start at the parking lot and take the steepest route to the top of the hill.

- a) Draw a contour map showing the height of land in the vicinity of the hill. On your map, draw and label a few contour lines, the trail and the parking lot.
- b) The (x, y) coordinates of each point on the trail obey $y = a(x + 1)^b$ for some constants a, b . Determine a and b .

[15] **3.** Let $f(x, y) = (4x^2 - y^2)e^{-x^2 - y^2}$.

- a) Find the maximum and minimum values of $f(x, y)$ on the xy -plane.
- b) Find the maximum and minimum values of $f(x, y)$ on the pie-shaped region $0 \leq y \leq 3x$.

- [15] 4. Find the points on the ellipsoid $x^2 + y^2 + z^2 + xy + yz = 24$ which are farthest from the xy -plane.

- [10] 5. Reverse the order of integration of

$$\int_0^4 dy \int_0^{\sqrt{y}} dx \frac{e^x}{4-x^2} + \int_4^8 dy \int_0^{\sqrt{8-y}} dx \frac{e^x}{4-x^2}$$

and then evaluate the integral.

- [10] **6.** Express the area below the curve $r = e^\theta$, $0 \leq \theta \leq \pi$ and above the x axis as a double integral in polar coordinates. Give an equivalent double integral with the order of integration reversed. Evaluate the area.

- [10] 7. Evaluate $\iiint_V z^2 dx dy dz$, where V is the part of the region inside $x^2 + y^2 + z^2 = a^2$ that satisfies $x \geq 0$, $y \geq 0$ and $z \geq \sqrt{\frac{x^2+y^2}{3}}$.

[15] 8. Consider the function

$$f(x, y) = \begin{cases} \frac{xy[e^x - 1]}{x^2 + 2y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

- a) Prove that f is continuous at $(0, 0)$.
b) Let D_ε be the disk $\{ (x, y) \mid x^2 + y^2 \leq \varepsilon^2 \}$. Evaluate

$$\lim_{\varepsilon \rightarrow 0} \frac{1}{A(D_\varepsilon)} \iint_{D_\varepsilon} [e^{f(x,y)} + 1] \, dx dy$$

where $A(D_\varepsilon)$ is the area of D_ε , giving a careful justification for your answer.

The End