FINAL EXAM

Math 217 Section 101

December 12, 2008

Last Name: ______________________________

First Name: ______________________________

Student Number: ______________________________

Signature: ______________________________

The exam is worth a total of 100 points with a duration of 2.5 hours. No books, notes or calculators are allowed. Justify all answers, show all work and **explain your reasoning carefully**. You will be graded on the clarity of your explanations as well as the correctness of your answers.

UBC Rules governing examinations:

(1) Each candidate should be prepared to produce his/her library/ AMS card upon request.
(2) No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in the examination questions.
(3) Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination, and and shall be liable to disciplinary action:
a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
b) Speaking or communicating with other candidates.
c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness will not be received.
(4) Smoking is not permitted during examinations.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Bonus	
Total	

1.) (14 points)
Find the maximum and minimum values of

$$f(x, y) = e^{-2x^2-y^2}(x^2 + 2y^2)$$

on the set $D = \{(x, y)|2x^2 + y^2 \leq 4\}$.

2.) (14 points)
Let $\hat{F}(x,y,z) =< 0, e^y \sin(x), e^z \sin(x) >$.
a) Find the curl of $\hat{F}$.
b) Find a function $h(x,y,z)$ such that the vector field

$$\hat{G}(x,y,z) =< h(x,y,z), e^y \sin(x), e^z \sin(x) >$$

is conservative. Find a function $g(x,y,z)$ such that $\hat{G}(x,y,z) = \nabla g(x,y,z)$.
c) Calculate $\int_C \hat{G} \cdot d\hat{r}$, where C is parametrized by $x(t) = t$, $y(t) = \sin(t)$ and $z(t) = \sin(t)$ for $0 \le t \le \frac{\pi}{2}$.
d) Find $\int_C \hat{F} \cdot d\hat{r}$, where C is given as in c). (Hint: Use the results from b) and c))

3.) (14 points)
Find the maximum and minimum values of $f(x, y, z) = 4x - 2y$ subject to the constraints $2z - x - y = 4$ and $x^2 + z^2 = 1$.

4.) (16 points)
Use the divergence theorem to calculate the flux of

$$\hat{F}(x,y,z) =< x(y+y^2) - \frac{x^3}{3}, x^2y - \frac{1}{2}y^2 - \frac{y^3}{3}, z + (x^3+1)y >$$

across the surface S given by $z = 4 - x^2 - y^2$, $z \geq 0$.

5.) (14 points)
Evaluate the following integrals
a) $\int_0^1 \int_{x^2}^1 x^3 e^{2y^3} \, dydx$
b) $\int_0^8 \int_{\sqrt[3]{y}}^2 3\sqrt{x^4+9} \;\; dxdy.$

6.) (14 points)
Let S be the part of the plane $x + y + z = 1$ that lies in the first octant (that is the region where $x \geq 0$, $y \geq 0$ and $z \geq 0$), and let C be the boundary of S oriented counterclockwise when viewed from above. Moreover let

$$\hat{F}(x, y, z) =< y\cos(2\pi x), e^x + 2, 1 - y(z-1) > .$$

a) Find the curl of $\hat{F}$.
b) Use Stokes' theorem to evaluate $\int_C \hat{F} \cdot d\hat{r}$.

7.) (14 points)
Evaluate

$$\int\int\int_E \frac{e^z}{\sqrt{4-x^2-y^2}}dV,$$

where E is the subset of the solid sphere $x^2+y^2+z^2 \le 4$ which lies inside the cylinder $x^2+y^2=1$ and above the xy-plane.

Bonus) (+10 points)
Use the change of variables formula and the transformation $x = u^2$, $y = v^2$, $z = w^2$ to find the volume of the region bounded by the surface

$$\sqrt{x} + \sqrt{y} + \sqrt{z} = 1, \quad x \geq 0, \quad y \geq 0, \quad z \geq 0$$

and the coordinate planes.