Be sure this exam has 12 pages including the cover

The University of British Columbia

MATH 215/255, Sections 101–105

Final Exam – December 2009

Name ______________________ **Signature** ______________________

Student Number ______________________

Circle Section: **101** Liu **102** Tsai **103** Seon **104** Dridi **105** Smith

No notes nor calculators.

1. Each candidate should be prepared to produce his library/AMS card upon request.

2. Read and observe the following rules:

No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination.

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Making use of any books, papers or memoranda, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates.

The plea of accident or forgetfulness shall not be received.

3. Smoking is not permitted during examinations.

problem	max	score
1.	12	
2.	10	
3.	13	
4.	10	
5.	20	
6.	10	
7.	10	
8.	15	
total	100	

(12 points) 1. Find the general solution of each of the following differential equations.

(a) $-y' + y = 4e^{3x}$.

(b) $y' = 4x(y-1)^{1/2}, \quad y > 1.$

(10 points) 2. Solve the initial value problem

$$(6xy - y^3)dx + (4y + 3x^2 + bxy^2)dy = 0, \quad y(0) = 3,$$

where b is a constant such that the differential equation is exact.

(13 points) 3. Use the method of variation of parameters to find the general solution of the given differential equation. Then check your answer by using the method of undetermined coefficients :

$$y'' + 2y' + y = 3e^{-t}$$

(10 points) 4. A mass of 1 g stretches a spring 5 cm. Suppose that the mass is also attached to a viscous damper with a damping constant of 20 dyn.s/cm. If the mass is pulled down an additional 2 cm and then released, find its position u at any time t.

(In CGS, the force unit is 1 dyn = 1 g$\cdot$cm/s^2, and the standard gravity is $g = 980$ cm/s^2. *Use* $384 = 6*64$).

5. Solve the following initial value problems.

(10 points) (a)

$$\begin{cases} y'' + y &= u_{\pi/2} + \delta(t - \pi/6) \\ y(0) = y'(0) &= 0 \end{cases}$$

(10 points) (b)

$$\begin{cases} y'' + 4y &= \frac{1}{k}\left[(t-5)u_5(t) - (t-5-k)u_{5+k}(t)\right] \\ y(0) = y'(0) &= 0 \end{cases}$$

(10 points) 6. For the linear system

$$\frac{d}{dt}\mathbf{X} = \begin{bmatrix} 0 & 4 \\ 1 & 0 \end{bmatrix} \mathbf{X},$$

find the general solution and plot a few trajectories.

(10 points) 7. Find the general solution of the following system.

$$\frac{d}{dt}\mathbf{X} = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}\mathbf{X} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^t.$$

8. Consider the nonlinear system

$$\frac{dx}{dt} = x(y-1), \qquad \frac{dy}{dt} = 2 + x - y. \tag{1}$$

(3 points) (a) Find the Jacobian (partial derivative) matrix for the system (1).

(4 points) (b) Find the critical points for the system (1).

(8 points) (c) Use the linear systems near each critical point to classify them, specifying both the type of critical point and whether it is (asymptotically) stable or unstable. If a critical point cannot be unambiguously classified using the linear systems, please indicate this in your answer.

Table of Laplace transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > 0$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. t^n, n positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$
4. t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$
5. $\sin(at)$	$\frac{a}{s^2+a^2}, \quad s > 0$
6. $\cos(at)$	$\frac{s}{s^2+a^2}, \quad s > 0$
7. $\sinh(at)$	$\frac{a}{s^2-a^2}, \quad s > \vert a\vert$
8. $\cosh(at)$	$\frac{s}{s^2-a^2}, \quad s > \vert a\vert$
9. $e^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
10. $e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
11. $t^n e^{at}$, n positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
14. $e^{ct}f(t)$	$F(s-c)$
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$
16. $\int_0^t f(t-\tau)g(\tau)d\tau$	$F(s)G(s)$
17. $\delta(t-c)$	e^{-cs}
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \ldots - f^{(n-1)}(0)$
19. $(-t)^n f(t)$	$F^{(n)}(s)$