THE UNIVERSITY OF BRITISH COLUMBIA
Sessional Examinations – April 2009

MATHEMATICS 215

TIME: 2.5 hours
NO AIDS ARE PERMITTED. Note that the maximum number of points is 110. A score of N/110 will be treated as N/100. Also note that this exam has **three** pages.

(15) 1. Consider the differential equation $\dfrac{dy}{dx} = 2x - y.$ (*)

(a) Find the general solution of (*).
(b) Find a particular solution (is there only one?) of (*) satisfying the initial condition $y(0) = -2$.
(c) Sketch the solution curves of (*).

(10) 2. Find the inverse function $f(t)$ of the Laplace transform for

(a) $F(s) = \dfrac{s}{s^2 - s - 2}$; (b) $F(s) = \dfrac{1}{s^2}\left[e^{-s} - (s^2+s)e^{-2s}\right]$.

In each case, evaluate $f(3)$.

(15) 3. Solve the initial value problem: $y'' + 4y = \cos t, \quad y(0) = y'(0) = 0.$

(15) 4. Solve the following initial value problem for $t > 0$ and sketch its solution:

$$y'' - y' = \delta(t-\pi), \quad y(0) = 1,\ y'(0) = 0\ .$$

[Note that $\delta(t)$ is the Dirac delta function.]

(10) 5. Find the general solution of each of the following differential equations:

(a) $y'' + 2y' + y = 0$; (b) $\dfrac{d^4y}{dx^4} + 4\dfrac{d^2y}{dx^2} = 0.$

(25) 6. (a) Solve the initial value problem

$$\frac{dx}{dt} = x - 3y,$$
$$\frac{dy}{dt} = 3x + 7y,$$

with $x(0) = 0,\ y(0) = 1.$

(b) Sketch the trajectory of the solution of (a) in the xy-phase plane for $-\infty < t < \infty$, , indicating by arrows the direction of increasing t.

(c) Solve the initial value problem

$$\frac{dx}{dt} = x - 3y + 1,$$
$$\frac{dy}{dt} = 3x + 7y + 1,$$

with $x(0) = 0,\ y(0) = 1.$

(20) 7. Consider the system

$$\frac{dx}{dt} = -y(y-2),$$

$$\frac{dy}{dt} = (x-2)(y-2),$$

for $t > 0$.

(a) Sketch the $y(t)$ component of the solution of this system for each of the following two sets of initial conditions:

(i) $x(0) = 1$, $y(0) = 2$;

(ii) $x(0) = 1$, $y(0) = 3$.

(b) Suppose one has the initial condition $x(0) = \alpha$, $y(0) = \beta$. Find all values of α and β for which

$\lim_{t\to\infty} x(t) = A$ and $\lim_{t\to\infty} y(t) = B$ both hold for some constants A and B.

(c) Find A and B.

TABLE OF INFORMATION

FUNCTION	**LAPLACE TRANSFORM**
$f(t)$	$F(s)$
$f'(t)$	$sF(s) - f(0)$
$u_a(t)$	$\frac{e^{-as}}{s}$
$u_a(t) f(t-a)$	$e^{-as}F(s)$
$\sin t$	$\frac{1}{s^2+1}$
$\cos t$	$\frac{s}{s^2+1}$
$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$
$tf(t)$	$-F'(s)$
$\int_0^t f(\tau)g(t-\tau)d\tau$	$F(s)G(s)$
$\delta(t-a)$	e^{-as}
$e^{at} f(t)$	$F(s-a)$