Math 215 FINAL EXAM
April 21, 2006

Last Name: _____

First Name: _____

Student Number: _____

Section Number: _____

Signature: _____

The exam is worth a total of 100 points with duration 2.5 hours. **No books, notes or calculators are allowed**. Justify all answers, show all work and explain your reasoning carefully. You will be graded on the clarity of your explanations as well as the correctness of your answers.

UBC Rules governing examinations:

(1) Each candidate should be prepared to produce his/her library/AMS card upon request.

(2) No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of the examination. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in the examination questions.

(3) Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination, and and shall be liable to disciplinary action:
a) Making use of any books, papers or memoranda, other than those authorized by the examiners.
b) Speaking or communicating with other candidates.
c) Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness will not be received.

(4) Smoking is not permitted during examinations.

Problem	Points
1	
2	
3	
4	
5	
6	
7	
Total	

Problem 1 (10 points). Solve the first order differential equation $xy' = (1 - 2x^2) \tan y$.

Problem 2 (10 points). Solve the first order differential equation

$$\left(x + \frac{2}{y}\right) dy + y \, dx = 0.$$

Problem 3 (15 points). Solve the 2nd order equation $y'' + 4y = x \sin x$.

Problem 4 (15 points). Solve the 2nd order equation $y'' + 2y' + y = e^{-x} \ln x$.

Problem 5 (15 points). Solve the initial value problem

$$y'' + 9y = f(t), \quad y(0) = 0, \quad y'(0) = 0$$

where the function f is given by

$$f(t) = \begin{cases} t, & 0 \leq t < 1 \\ 0, & t \geq 1 \end{cases}$$

Problem 6 (15 points). Solve the initial value problem

$$\frac{dx}{dt} = x - 2y + 1$$
$$\frac{dy}{dt} = x - y$$

with $x(0) = 3, y(0) = 2$.

Problem 7 (20 points). Consider the nonlinear system

$$\frac{dx}{dt} = 2x(2-y)$$
$$\frac{dy}{dt} = 3y(3-x).$$

(a) Find the critical points of the system.

(b) Sketch the phase portrait of the linearization of the system at each critical point. Plot the eigendirections carefully, and specify the type (node, saddle, spiral, center; stable, unstable).

(c) Sketch the phase portrait of this nonlinear system.

$$\frac{dx}{dt} = 2x(2-y)$$