Marks

[10] **1.** Let L be a line which is parallel to the plane $2x + y - z = 5$ and perpendicular to the line $x = 3 - t$, $y = 1 - 2t$ and $z = 3t$.

a) Find a vector parallel to the line L.

b) Find parametric equations for the line L if L passes through a point $Q(a, b, c)$ where $a < 0, b > 0, c > 0$, and the distances from Q to the xy-plane, the xz-plane and the yz-plane are $2, 3$ and 4 respectively.

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[10] **2.** Let $z = f(x, y) = ln(4x^2 + y^2)$

(a) Use a linear approximation of the function $z = f(x, y)$ at $(0, 1)$ to estimate $f(0.1, 1.2)$

(b) Find a point $P(a, b, c)$ on the graph of $z = f(x, y)$ such that the tangent plane to the graph of $z = f(x, y)$ at the point P is parallel to the plane $2x + 2y - z = 3$

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[10] **3.** Let $z = f(x, y)$, where $f(x, y)$ has continuous second-order partial derivatives, and

$$x = 2t^2, y = t^3, f_x(2,1) = 5, f_y(2,1) = -2, f_{xx}(2,1) = 2, f_{xy}(2,1) = 1, f_{yy}(2,1) = -4.$$

Find $\dfrac{d^2z}{dt^2}$ when $t = 1$.

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[10] **4.** The temperature at a point (x, y, z) is given by $T(x, y, z) = 5e^{-2x^2-y^2-3z^2}$, where T is measured in centigrade and x, y, z in meters.

(a) Find the rate of change of temperature at the point $P(1, 2, -1)$ in the direction toward the point $(1, 1, 0)$.

(b) In which direction does the temperature decrease most rapidly?

(c) Find the maximum rate of decrease at P.

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[10] **5.** Let C be the intersection of the plane $x + y + z = 2$ and the sphere $x^2 + y^2 + z^2 = 2$.

(a) Use Lagrange multipliers to find the maximum value of $f(x, y, z) = z$ on C

(b) What are the coordinates of the lowest point on C?

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[10] **6.** (a) Combine the sum of the iterated integrals

$$I = \int_0^1 \int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)dxdy + \int_1^4 \int_{y-2}^{\sqrt{y}} f(x,y)dxdy$$

into a single iterated integral with the order of integration reversed.

(b) Evaluate I if $f(x,y) = \dfrac{e^x}{2-x}$.

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[10] **7.** The average distance of a point in a plane region D to a point (a, b) is defined by

$$\frac{1}{A(D)}\iint_D \sqrt{(x-a)^2+(y-b)^2}\ dxdy$$

where $A(D)$ is the area of the plane region D. Let D be the unit disk $1 \geq x^2 + y^2$. Find the average distance of a point in D to the center of D.

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[10] **8.** Let E be the region in the first octant bounded by the coordinate planes, the plane $x + y = 1$ and the surface $z = y^2$.

Evaluate $\displaystyle\iiint_E z\,dV$.

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[10] **9.** Let E be the smaller of the two solid regions bounded by the surfaces $z = x^2 + y^2$ and $x^2 + y^2 + z^2 = 6$.

Evaluate $\displaystyle\iiint_E (x^2 + y^2)\,dV$.

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[10] **10.** Evaluate $I = \int\int\int_{R^3} [1 + (x^2 + y^2 + z^2)^3]^{-1} dV$.

The End

Be sure that this examination has 11 pages including this cover

The University of British Columbia

Final Examination - April, 2012

Mathematics 200

Closed book examination

Time: 2 $\frac{1}{2}$ hours

Name ____________________ **Signature** ____________________

Student Number____________________ **Instructor's Name** ____________________

Section Number ____________________

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