The University of British Columbia
Final Examination - December 7th, 2007

Mathematics 200, joint final

Closed book examination — Time: 3.0 hours

Name ______________________ **Signature** ______________________

Student Number ______________

Special Instructions:
- Be sure that this examination has 12 pages. Write your name on top of each page.
- No calculators or notes are permitted.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

- Each candidate should be prepared to produce her/his library/AMS card upon request.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of examination.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Making use of any books, papers, or memoranda, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates.

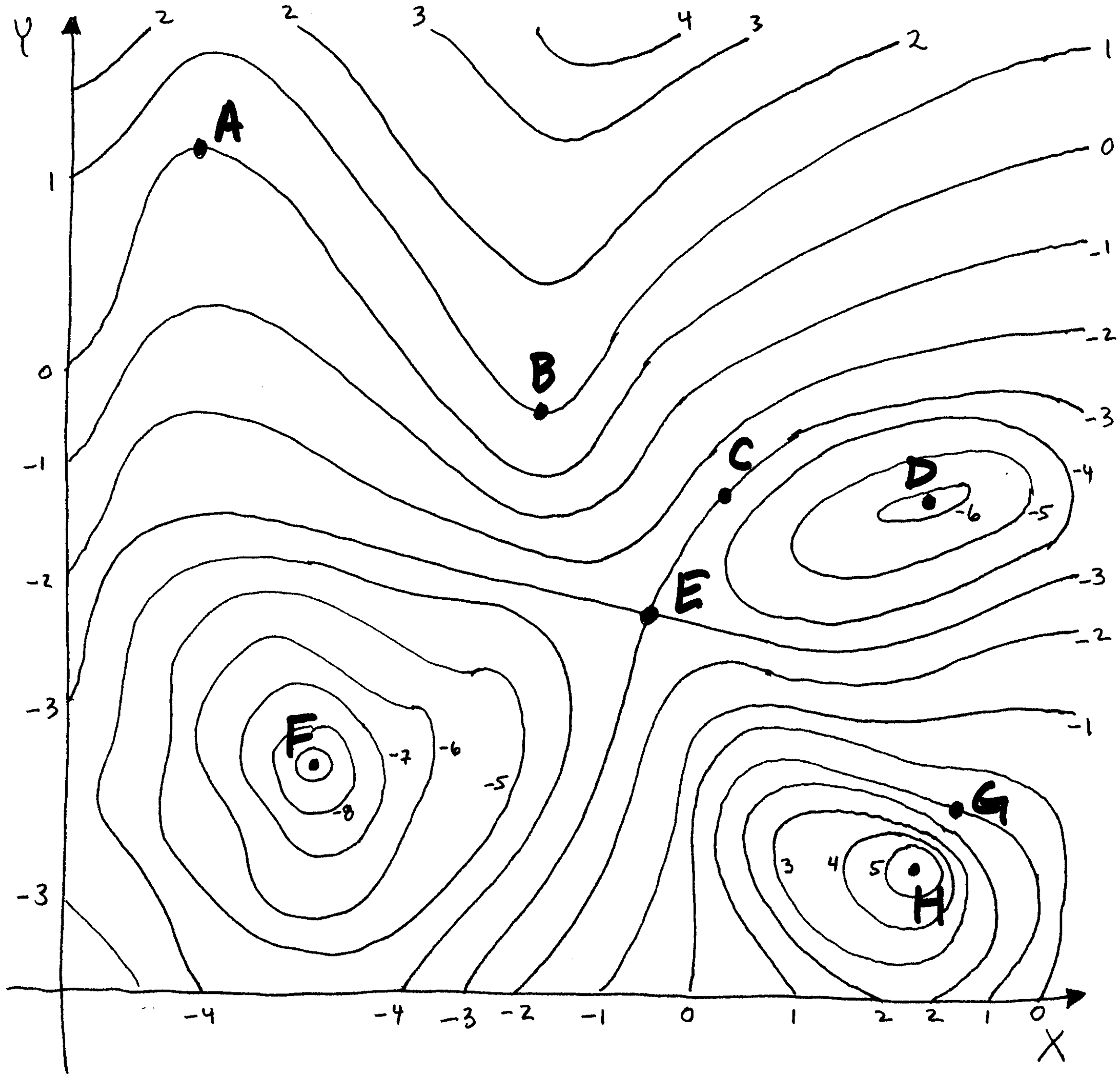
1		11
2		12
3		11
4		11
5		11
6		11
7		11
8		11
9		11
Total		100

Problem 1. (11 points.)
Let A and B be the points with coordinates $(1, 2, 3)$ and $(-1, 5, 1)$ respectively.

1. **(3 points)** Find symmetric equations for the line L passing through A and B.
2. **(3 points)** Let C be the point $(5, 2, -1)$. Find the area of the triangle ABC.
3. **(3 points)** Find the angle between the sides AB and BC in the triangle ABC. You may express your answer in terms of arccos.
4. **(2 points)** Find an equation for the plane passing through C and perpendicular to L.

Problem 2. (12 points) Consider a twice differentiable function $f(x, y)$ illustrated by the contour map on the follow page. (The numbers on the contour plot give the function values along the contours.)

1. (**2 Points.**) Draw the direction of ∇f at point C on the diagram.
2. (**2 Points.**) Which of the 8 points in the diagram (A-H) are critical points? Classify these points as local minima, local maxima, or saddle points.
3. Identify each of the following statements as true or false. **2 points will be given each correct answer, -2 points for each incorrect answer, 0 points for no answer.**
   (a) The derivative of f at the point C, in the direction $\mathbf{u} = \langle -2, -1 \rangle$ is positive.
   (b) $\frac{dy}{dx}$ at the point G along the level curve $f(x, y) = 1$ is negative.
   (c) f_y at the point G is positive.
   (d) f_{yy} at the point G is positive.



Y
X
A
B
C
D
E
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G
H
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0

Problem 3. (11 points.)
Suppose that the temperature of a metal plate is given by $T(x, y) = e^{xy}$, for points (x, y) on the elliptical plate defined by $(x - 2)^2 + 2y^2 \leq 12$. Find the maximum and minimum temperatures on the plate.

Problem 4. (11 points.)
Evaluate the integral

$$\int_{x=0}^{x=1} \int_{y=x^2}^{y=1} \sqrt{y}\, e^{y^2}\, dy\, dx.$$

Problem 5. (**11 points.**)
Let E be the solid bounded by the graphs of $z = 4 - y^2$, $x + z = 4$, $x = 0$, and $z = 0$.

1. **(4 points)** Express the triple integral $\iiint_E f dV$ as an iterated integral in the following order (do not evaluate)

$$\int_{y=}^{y=} \int_{z=}^{z=} \int_{x=}^{x=} f(x, y, z)\, dx\, dz\, dy$$

2. **(4 points)** Express the triple integral $\iiint_E f dV$ as an iterated integral in the following order (do not evaluate)

$$\int_{x=}^{x=} \int_{z=}^{z=} \int_{y=}^{y=} f(x, y, z)\, dy\, dz\, dx$$

3. **(3 points)** Find the volume of E.

Problem 6. (11 points.)
Find and classify all critical points of the function

$$f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2.$$

Problem 7. (**11 points.**) The total resistance R, of three resistors connected in parallel, is a function of the individual resistances x, y, and z. The function R is determined by the relation

$$\frac{1}{R} = \frac{1}{x} + \frac{1}{y} + \frac{1}{z}.$$

1. **(5 points)** Find R_x, R_y, and R_z.
2. **(6 points)** The resistances are measured in ohms as $x = 25$, $y = 40$, and $z = 50$, with a possible error of 0.5% in each case. Estimate the maximum error in the calculated value of R.

Problem 8. (**11 points.**) Let E be the solid region bounded by the cylinders $x^2 + y^2 = 1$ and $x^2 + z^2 = 1$.

1. Find the volume of E.
2. Find the area of the boundary surface of E.

Problem 9. (**11 points.**) A metal plate occupies the region R which lies inside the circle $x^2 + y^2 = 2y$ but outside the circle $x^2 + y^2 = 1$. The plate has mass density given by $\rho(x, y) = 1/\sqrt{x^2 + y^2}$.

1. **(3 points)** Evaluate the integral

$$\iint_R \rho(x, y)\, dx\, dy$$

2. **(3 points)** Evaluate the integral

$$\iint_R x\rho(x, y)\, dx\, dy$$

3. **(3 points)** Evaluate the integral

$$\iint_R y\rho(x, y)\, dx\, dy$$

4. **(2 points)** Find $(\overline{x}, \overline{y})$, the center of mass of the plate.

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