The University of British Columbia
Final Examination - April 26, 2005

Mathematics 200
Instructors: Jim Bryan and Joseph Lo

Closed book examination Time: 2.5 hours

Name ______________________ **Signature** ______________________

Student Number ______________

Special Instructions:
- Be sure that this examination has 15 pages. Write your name on top of each page.
- No calculators or notes are permitted.
- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

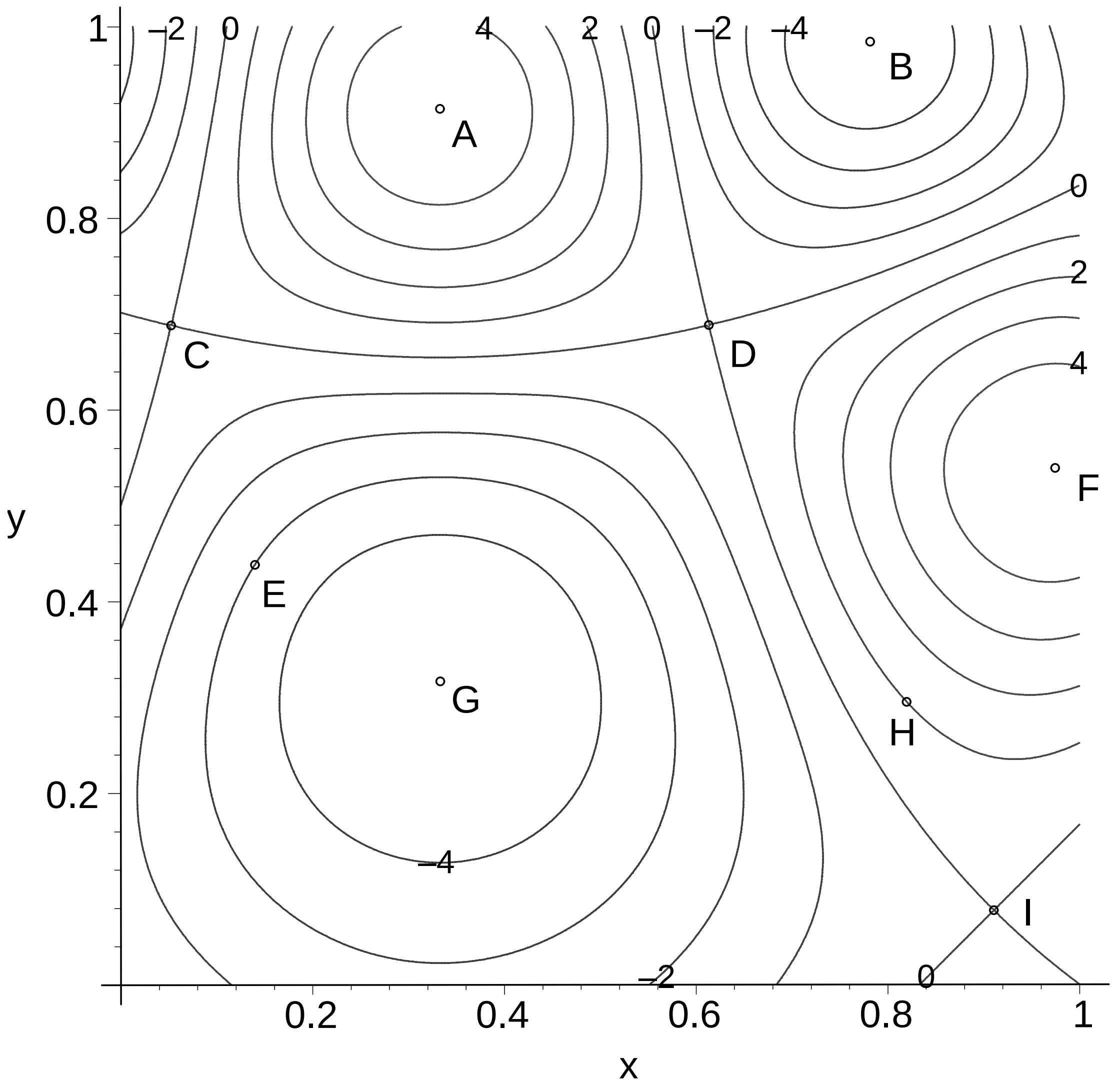
Rules governing examinations

• Each candidate should be prepared to produce her/his library/AMS card upon request.
• No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of examination.
• Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
• CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
(a) Making use of any books, papers, or memoranda, other than those authorized by the examiners.
(b) Speaking or communicating with other candidates.
(c) Purposely exposing written papers to the view of other candidates.

1		14
2		10
3		6
4		6
5		10
6		12
7		10
8		10
9		12
10		10
Total		100

Problem 1. Consider a twice differentiable function $f(x, y)$ illustrated by the contour map on the follow page.

1. (**3 Points.**) Draw the direction of ∇f at point H on the diagram.
2. (**3 Points.**) Which of the 9 points in the diagram (A-I) are critical points? Classify these points as local minima, local maxima, or saddle points.
3. (**2 Points each.**)State whether the following quantities at point E are positive or negative.
   (a) derivative of f in the direction $\mathbf{u} = \langle -2, 1 \rangle$
   (b) $\frac{dy}{dx}$ along the level curve $f(x, y) = -3$
   (c) f_y
   (d) f_{yy}



1
0.8
0.6
0.4
0.2
y
0.2
0.4
0.6
0.8
1
x
A
B
C
D
E
F
G
H
I
–2
0
4
2
0
–2
–4
0
2
4
–4
–2
0

Problem 2. (**10 points.**) Find the global maximum and global minimum of the function $f(x, y) = 2x^2 + 3y^2 - 4x - 5$ on the disk $x^2 + y^2 \leq 16$.

Problem 3. (**6 points.**) Find the distance between the plane $x - 6y + 8z = 1$ and the line $x = 2y = 4z$.

Problem 4. (**6 points.**) Find the area of the triangle whose vertices are at $A(2, -2, 1)$, $B(3, -1, 2)$ and $C(5, 0, 3)$. Find a unit vector perpendicular to the plane ABC.

Problem 5. (**10 points.**) Find the equation of the tangent plane to the surface $x - z = 4\arctan(yz)$ at the point $(1 + \pi, 1, 1)$.

Problem 6. (**12 points.**) Find the average value of $f(x,y,z) = x^2(x^2+y^2)$ inside the region E bounded above by $z = 6 - 4(x^2+y^2)$ and below by $z = 2\sqrt{x^2+y^2}$. Simplify your answer.

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Problem 7. (12 points.) Suppose the triple integral over a region E is given by

$$\iiint_E f(x, y, z)\ dV = \int_{-1}^{1} \int_0^{\sqrt{1-y^2}} \int_0^x \left(x^2 + y^2\right)\ dz\, dx\, dy.$$

Rewrite the integral as an equivalent iterated integral in the order of

1. $dy\, dx\, dz$,
2. $dx\, dz\, dy$,
3. $dz\, dr\, d\theta$. (In this case, express everything in terms of r, θ, and z.)

Do not evaluate the integral.

Problem 8. (**10 points.**) Find the surface area of the part of the surface $z = \frac{2}{3}(x^{3/2} + y^{3/2})$ that lies over the square $0 \leq x \leq 1$, $0 \leq y \leq 1$.

Problem 9. (**12 points.**) Let X and Y be the time spent waiting for the train to Xanadu and the train to Yonkers respectively. X and Y are random variables that have a joint probability density function given by

$$f(x, y) = \begin{cases} 0 & \text{if } x < 0 \text{ or } y < 0 \\ 2e^{-(2x+y)} & \text{if } x \geq 0 \text{ and } y \geq 0 \end{cases}$$

1. Compute $\mu(X)$, the expected waiting time for the Xanadu train.
2. Compute $\mu(Y)$, the expected waiting time for the Yonkers train.
3. Compute the probability that the Xanadu train arrives before the Yonkers train.

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Problem 10. (**10 points.**) Evaluate the integral:

$$\int_0^1 \int_{x^2}^1 x^3 \sin(y^3)dy\, dx$$

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