Final Exam   Duration: 2.5 hours
*This test has **10 questions** on **15 pages**, for a total of 85 points.*

- Read all the questions carefully before starting to work.

- All questions are long-answer; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.

- Continue on the back of the previous page if you run out of space.

- Attempt to answer all questions for partial credit.

- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

First Name: _____ Last Name: _____

Student-No: _____

Signature: _____

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	8	8	8	19	6	10	7	7	6	6	85
Score:											

Justify your answers and show all your work. Unless otherwise indicated simplification of answers is not required.

1. Consider the following function

$$f(x) = 2x^4 + \frac{1}{x} - \sqrt[3]{x^2} + 3e.$$

4 marks   (a) Compute the derivative of the function $f(x)$.

4 marks   (b) Compute the general anti-derivative of the function $f(x)$.

4 marks   2.   (a) Find the equation of the tangent line to

$$f(x) = e^x \cos x$$

at the point $x = \frac{\pi}{2}$.

4 marks   (b) Find a function $g(x)$ satisfying $g(\pi/4) = 1$ such that

$$g'(x) = \sqrt{2} \sin x + 3 \cos (2x).$$

4 marks 3. (a) Find the derivative of

$$f(x) = \ln\left(\sin\left(x^2\right)\right).$$

4 marks (b) Compute

$$\int_{-1}^{3} g(x)\ dx$$

where

$$g(x) = \begin{cases} 1, & x > 0 \\ e^x, & x \leq 0 \end{cases}.$$

4. Compute the following integrals.

4 marks (a)

$$\int_1^4 \frac{\sqrt{x} + x^2}{x} \, dx$$

5 marks (b)

$$\int x^2 \sin\left(x^3 + 1\right) \, dx$$

5 marks (c)

$$\int (2x+1)\sin x \ dx$$

5 marks (d)

$$\int \ln x \ dx$$

6 marks 5. You are currently 3 meters East of a flag pole and move towards the pole at 1 meter per second. Your friend is 4 meters North of the pole and moves away from it at 2 meters per second. How fast is the distance between you and your friend changing? Is the distance increasing or decreasing?

4 marks 6. (a) Explain using a picture why the expression

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

should give exactly the area under the curve $f(x)$. Indicate on your picture what n, x_i, $f(x_i)$ and Δx are.

6 marks (b) Approximate the following integral using Riemann Sums

$$\int_1^{2.5} 2x \ dx.$$

Use left endpoints and $n = 3$ (ie. four bars). Is your approximation less than, greater than, or exactly equal to the true value of the integral? Explain why.

7. After Matt graduates he's going to spend the summer scooping ice cream cones. On his first day the *rate of change* of the number of ice cream cones he produces is

$$r(t) = 5 - \frac{4}{(t+1)^2}.$$

in units of cones/minute.

6 marks   (a) Assuming that at $t = 0$ he has produced no cones, how many cones will he have produced after his first 3 minutes?

1 mark   (b) After a long period of time Matt will get good at scooping ice cream. Eventually, about how many cones will he be able to produce per minute?

3 marks 8. (a) Suppose that

$$\int_{1}^{-1} f(x) \; dx = 4 \quad \text{and} \quad \int_{1}^{2} f(x) \; dx = 3.$$
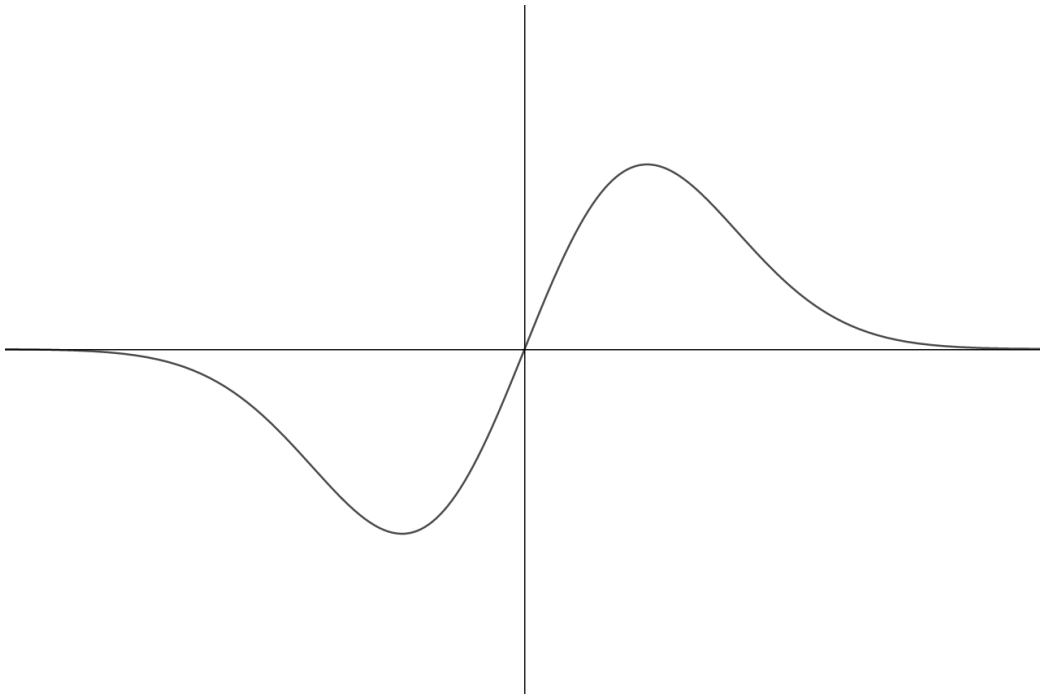
Find

$$\int_{-1}^{2} f(x) \; dx.$$

4 marks (b) Find *all* values of b such that

$$\int_{3}^{b} e^{7x} \; dx = 0.$$

Ensure you justify your answer fully.

9. Observe the following graph of $f(x)$

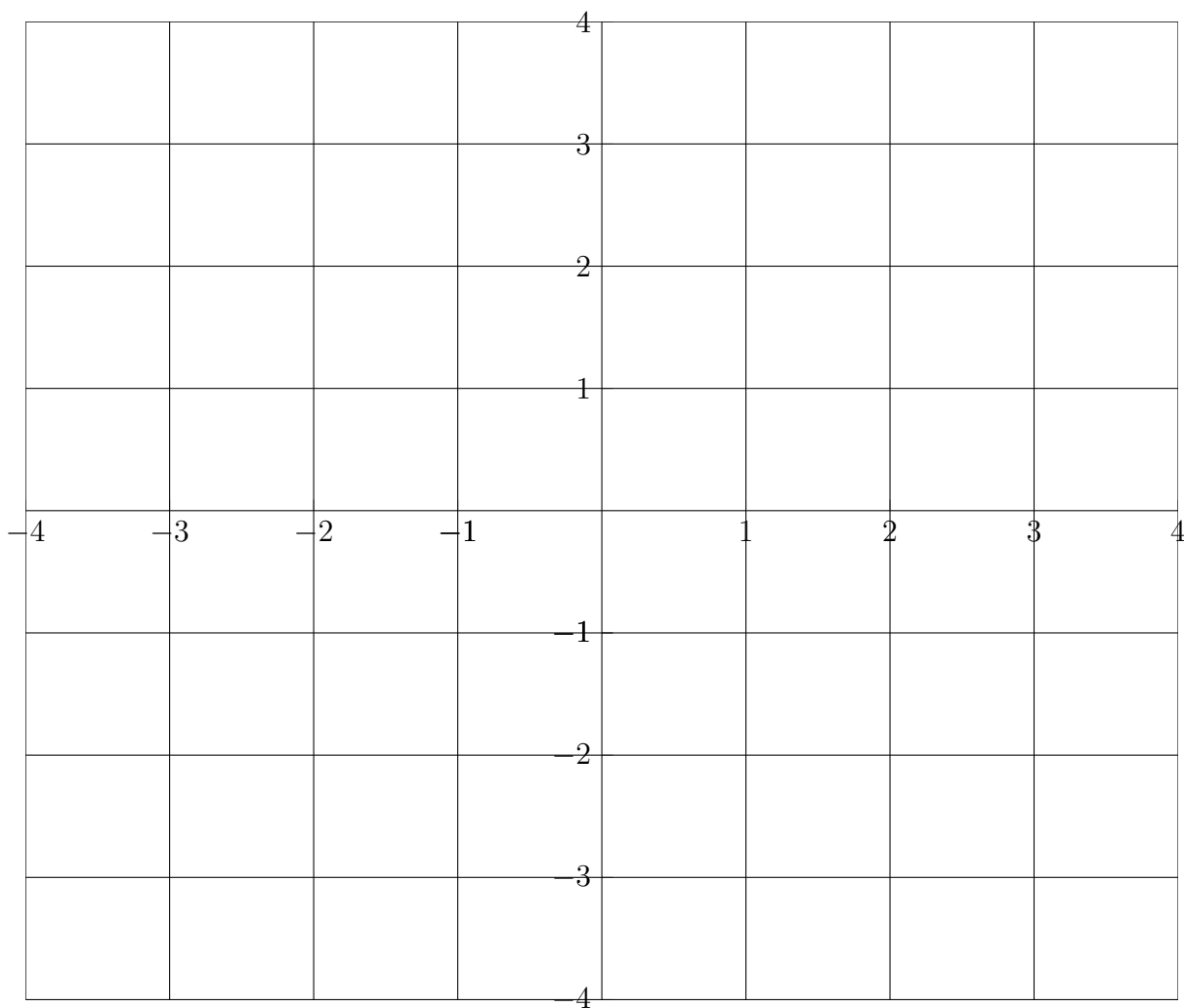1 mark   (a) Is $f(x)$ an even or odd function?

5 marks   (b) Sketch the graph of the derivative of $f(x)$. Is $f'(x)$ even or odd?

6 marks   10. Sketch the graph of a function with domain $f(x)$ is $\{x \in \mathbb{R} : -4 \leq x \leq 4\}$ satisfying the following properties:
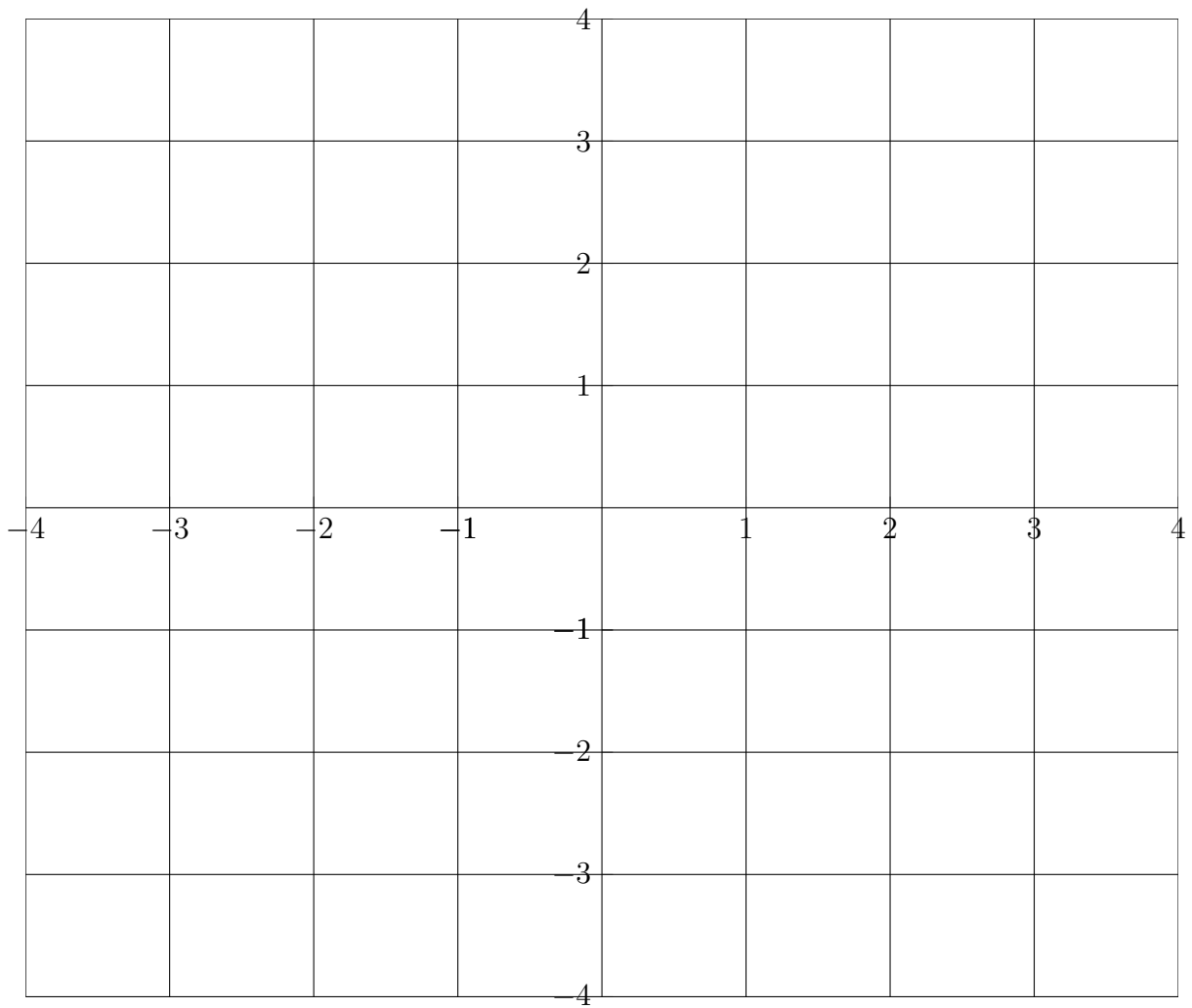
- $f(-3) > 0$
- $f(1) < 0$
- $f'(-2) = 0$
- $\displaystyle\int_{-4}^{0} f(x) \, dx = 0$
- $\displaystyle\int_{0}^{4} f(x) \, dx > 0$
- $\displaystyle\lim_{x \to 3} f'(x) = \infty$

You do not need to find an equation for your function. Use the axes below.

There is another set of a axes on the following page (in case you ruin the first one).

Extra axes:

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