The University of British Columbia

Final Examination - April 25, 2014

Mathematics 121

Time: 2.5 hours

FAMILY Name ______________________________

First Name __________________ **Signature** __________________

Student Number _______________

MATH 121 **Section Number:**__________

Special Instructions:

No memory aids are allowed. No communication devices allowed. No calculators allowed. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them. Calculator-ready form is acceptable for all numerical answers.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		33
2		6
3		5
4		5
5		6
6		4
7		6
8		7
9		8
Total		80

[33] **1**. **Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work. **Simplify your answers as much as possible.**

(a) Evaluate $\int x \ln x \, dx$.

Answer:

(b) Evaluate $\int \frac{x^2}{(x^3+1)^{101}} \, dx$.

Answer:

(c) Evaluate $\int \cos^3 x \sin^4 x \, dx$.

Answer:

(d) Evaluate $\int \sqrt{4 - x^2}\; dx$.

Answer:

(e) Consider the Trapezoid Rule for making numerical approximations to $\int_a^b f(x)\ dx$. The error for the Trapezoid Rule satisfies $|E_T| \leq \dfrac{K(b-a)^3}{12n^2}$, where $|f''(x)| \leq K$ for $a \leq x \leq b$. If $-6 < f''(x) < 0$ for $2 \leq x \leq 7$, find the smallest possible value of n to guarantee the Trapezoid Rule will give an approximation for $\int_2^7 f(x)\, dx$ with absolute error, $|E_T|$, less than 0.001.

Answer:

(f) Find the values of p for which the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ converges.

Answer:

(g) Find the radius of convergence for the power series $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$.

Answer:

(h) Find a power series representation for $\dfrac{x^3}{1-x}$.

Answer:

(i) Find the coefficient c_5 of the fifth degree term in the Maclaurin series $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ for e^{3x}.

Answer:

(j) Let $f(x) = \int_1^x 100(t^2 - 3t + 2)e^{-t^2}\, dt$. Find the interval(s) on which f is increasing.

Answer:

(k) Use series to evaluate $\lim_{x \to 0} \frac{1 - \cos(x)}{1 + x - e^x}$.

Answer:

Long Problems. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

[6] **2**. Find the volume of the solid generated by rotating the finite region bounded by $y = 1/x$ and $3x + 3y = 10$ about the x-axis. It will be useful to sketch the region first.

[5] **3**. Find the solution of the differential equation

$$x\frac{dy}{dx} + y = y^2$$

that satisfies $y(1) = -1$.

[5] **4**. Find the length of the polar curve $r = \theta^2$ for $0 \leq \theta \leq \pi$.

[6] **5.** Let R be the region satisfying $0 \le y \le \sin x$, $0 \le x \le \pi$.

(a) Find the area and centroid of R.

(b) Find the volume of the solid of revolution generated by rotating R about the y-axis.

[4] **6.** Carefully state what you consider to be the most important theorem in the course. Points will be given for good taste.

[6] **7**. Determine, with explanation, whether the following series converge or diverge.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{9^n n!}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^{\log n}}$

[7] **8.**

(a) Let f be an increasing function on $[0, 1]$ and set $D = f(1) - f(0)$. Let $x_i = i/n$, $R_n = \sum_{i=1}^{n} f(x_i)\frac{1}{n}$, and $S_n = \sum_{i=1}^{n} f(x_{i-1})\frac{1}{n}$. Find $R_n - S_n$ in terms of D. Use this to prove that f is integrable on $[0, 1]$.

(b) Use the expression for $R_n - S_n$ from (a) to find n so that $|\int_0^1 \sin(x^2\pi/6)dx - R_n| < .01$. Justify your answer carefully. You should find the smallest value of n you can, and must of course show the hypotheses from (a) are satisfied.

[8] **9**. Assume $\{a_n\}$ is a sequence such that na_n decreases to C as $n \to \infty$ for some real number $C > 0$

(a) Find the radius of convergence of $\sum_{n=1}^{\infty} a_n x^n$. Justify your answer carefully.

(b) Find the interval of convergence of the above power series, that is, find all x for which the power series in (a) converges. Justify your answer carefully.