

The University of British Columbia
Final Examinations - December 2011

MATHEMATICS 120

CLOSED BOOK EXAMINATION

Time: 3 hours

Notes, calculators are not permitted.

All seven questions are of equal value.

1. (a) Evaluate $\frac{d}{dx}(xe^{2x} \cos 4x)$.

(b) Find $f'(x)$ if $f(x) = x^x + \log_{10} x$.

(c) Evaluate $\lim_{x \rightarrow 0} \frac{\sin(x^{27} + 2x^5 e^{x^{99}})}{\sin^5 x}$.

2. Consider the curves given by the equation

$$y = f(x) = \frac{a}{x^2} + \frac{1}{x}, \quad a = \text{const}, \quad -\infty < a < \infty, \quad \text{defined on the region } R \text{ where } x > 0, y \geq 0. \text{ Note that the region } R \text{ may depend on the value of the constant } a.$$

Sketch all possible curves, indicating asymptotes, local maxima and minima, global maxima and minima, inflection points, and where the graphs are concave up/concave down. Note that the sketches may depend on the value of the constant a .

3. A length of wire is cut into two pieces, one of which is bent to form a circle, the other to form a square. How should the wire be cut if the area enclosed by the two curves is

(a) maximized?

(b) minimized?

Justify your answers.

4. (a) Sketch the graph of $y = f(x) = x^5 - x$, indicating asymptotes, local maxima and minima, inflection points, and where the graph is concave up/concave down.

(b) Consider the function $f(x) = x^5 - x + k$, where k is a constant, $-\infty < k < \infty$. How many roots does the function have? Your answer might depend on the value of k .

5. Use Newton's method as an aid to find each root of $f(x) = x^5 - x + \frac{4}{5}$ to within 10% accuracy. In particular express each root by a rational number accurate to within 10%. Justify each answer by a careful error analysis.
6. The equation $(\sin y)(\cosh x) = 1$, $0 < y \leq \frac{\pi}{2}$, defines y implicitly as a function of x , $-\infty < x < \infty$.
- (a) Express y' in terms of x and/or y .
- (b) Evaluate $\lim_{x \rightarrow 0^+} y'(x)$.
- (c) Graph the equation. Indicate any symmetry. Indicate where the slope is positive, negative. Indicate (with proof) where the graph is concave up/concave down. Find all local maxima and minima as well as global maximum and minimum values.
7. A particle moves on the path given by the parametric equations

$$x = t^3 - 3t^2, \quad y = t^2 - 3t, \quad 0 \leq t < \infty.$$

Sketch the path traced out by the particle. Find all asymptotes. Indicate where the tangent line to the path is horizontal or vertical. Indicate at what time(s) the path crosses, i.e., find the times t and $T \neq t$ when $(x(t), y(t)) = (x(T), y(T))$. Find the location of each crossing point. Indicate where the graph is concave up/concave down.