Name (print, with surname first):

Student number:

University of British Columbia
MATH 110: APRIL EXAM

Date: *April 22, 2013*
Time: *3:30 p.m. to 6:00 p.m.*
Number of pages: *13 (including cover page)*
Exam type: *Closed book*
Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

For examiners' use only		
Question	Mark	Possible marks
1		8
2		10
3		12
4		5
5		6
6		6
7		4
8		4
9		5
Total		60

1. Determine whether each of the following statements is true or false. If it is true, provide justification. If it is false, provide a counterexample.

   (a) [**2 marks**] If f is a polynomial of degree greater than 1, then f has a tangent line that crosses the x-axis.

   (b) [**2 marks**] If f has no critical points, then f has no inflection points.

   (c) [**2 marks**] If f has a local maximum at 1, then $f'(1) = 0$.

   (d) [**2 marks**] If f has a vertical asymptote at $x = 0$, then its reciprocal $1/f$ has a vertical asymptote at $x = 0$.

2. For each of the parts below, sketch a single function satisfying all of the listed criteria. Note that this question continues onto the next page.

(a) [**3 marks**] $f(3) = 0$, $f'(3) < 0$, and $f''(3) > 0$.
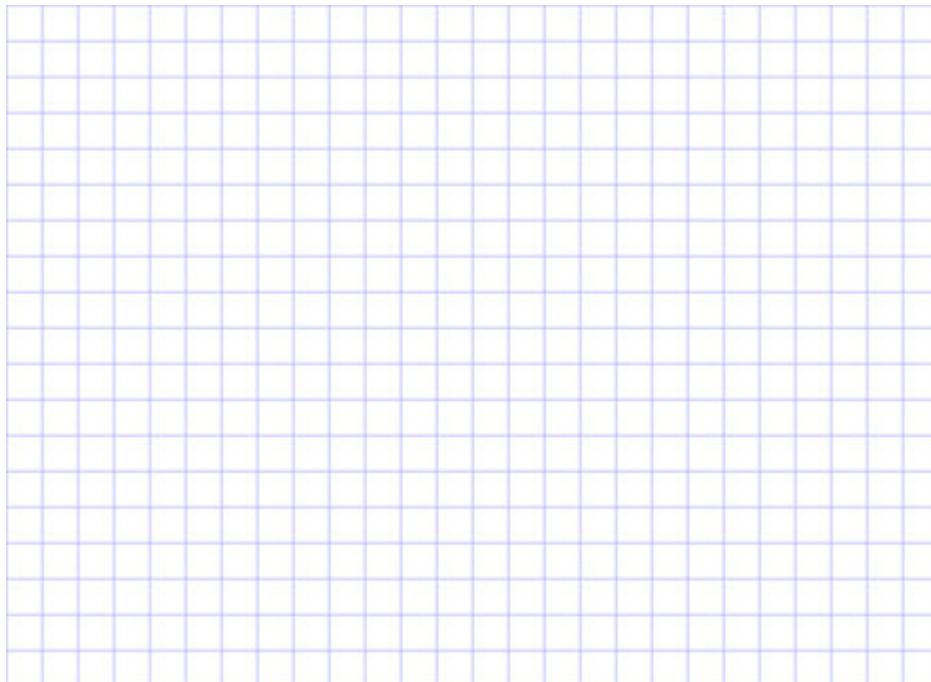
(b) [**3 marks**] $\lim\limits_{x \to -1^-} f(x) = 0$, $\lim\limits_{x \to -1^+} f(x) = \infty$, and $\lim\limits_{x \to 1} f(x) = -\infty$.
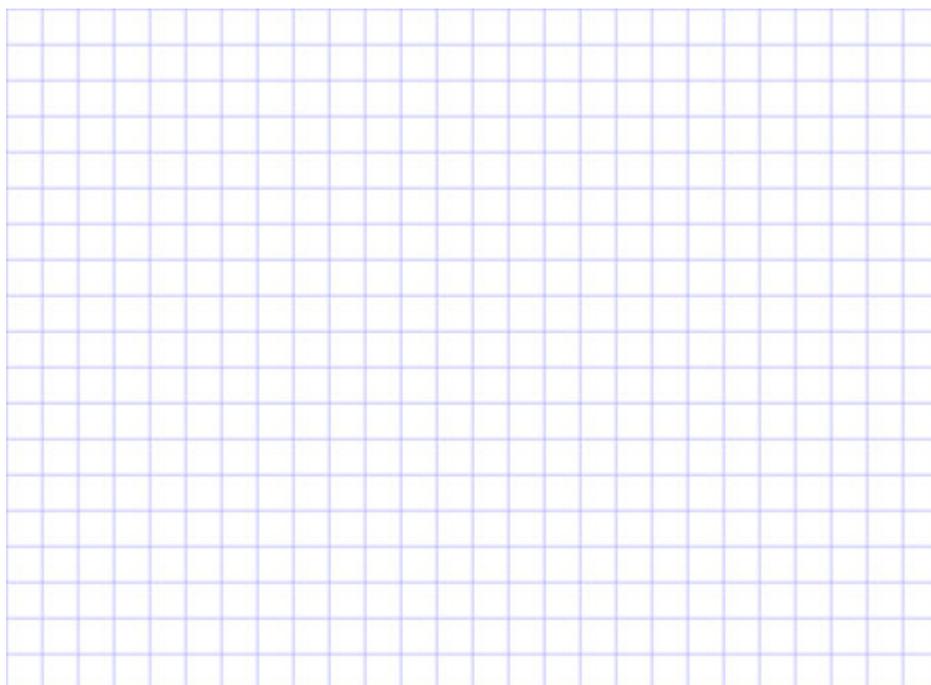
(c) **[2 marks]** f has an inflection point at $x = 1$ and a critical point at $x = 2$.
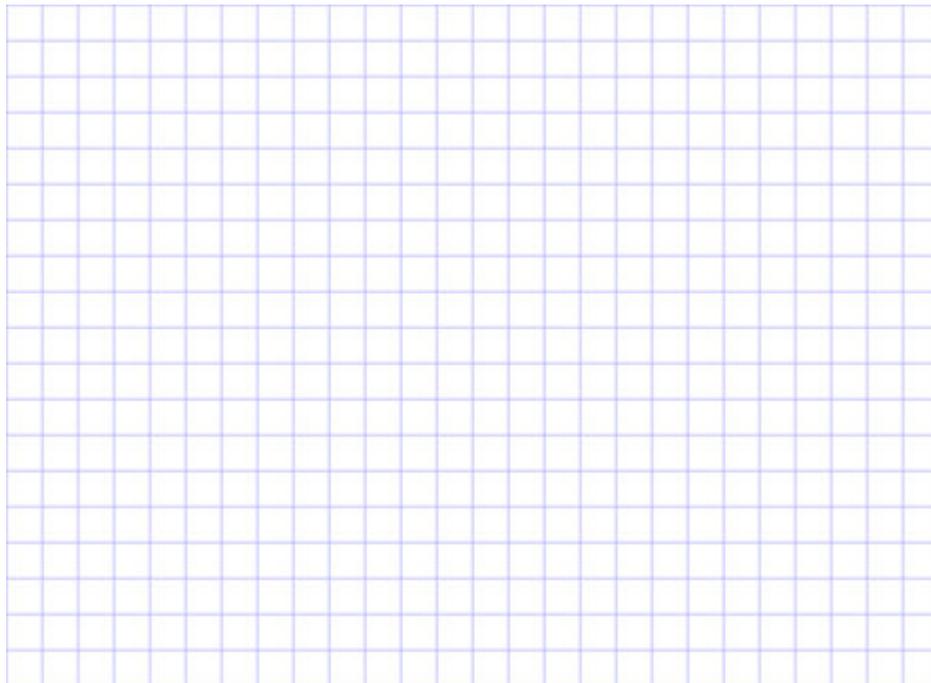
(d) **[2 marks]** $f'(x) = kf(x)$ for some positive constant k.
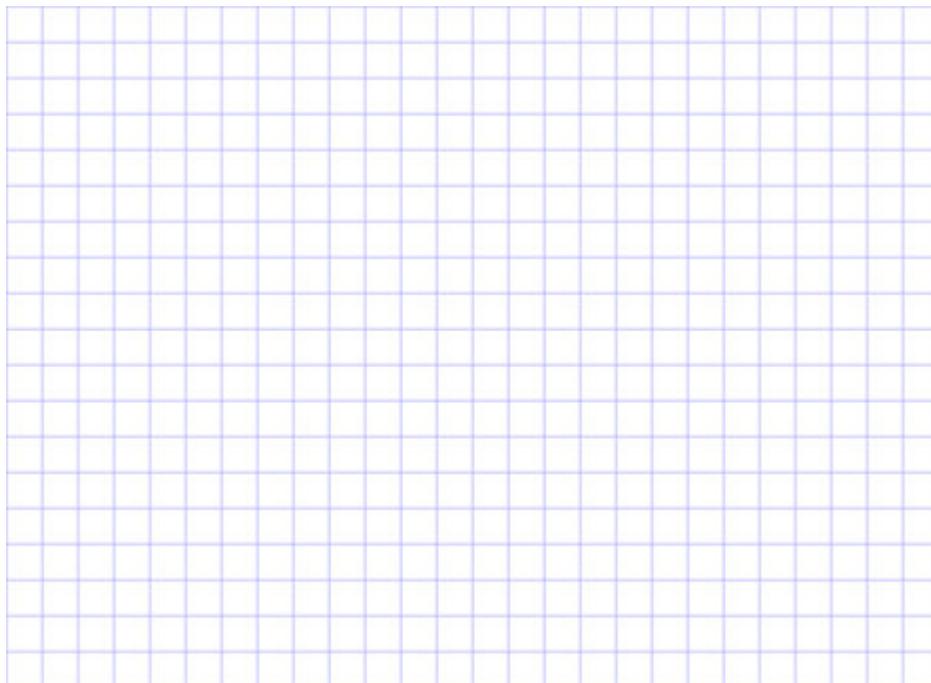
3. Let $f(x) = \dfrac{\sin x}{e^x}$ on the interval $[0, \pi]$. In this question you will sketch this function.

(a) **[1 mark]** Find the x-intercepts of f, if any exist.

(b) **[1 mark]** Find the vertical asymptotes of f, if any exist.

(c) **[4 marks]** Determine where f is increasing and where it is decreasing. Does f have any local extrema?

(d) [**4 marks**] Determine where f is concave up and where it is concave down. Does f have any inflection points?

(e) [**2 marks**] Make a large sketch of the graph of f below, including all you have found in the previous parts of the question.
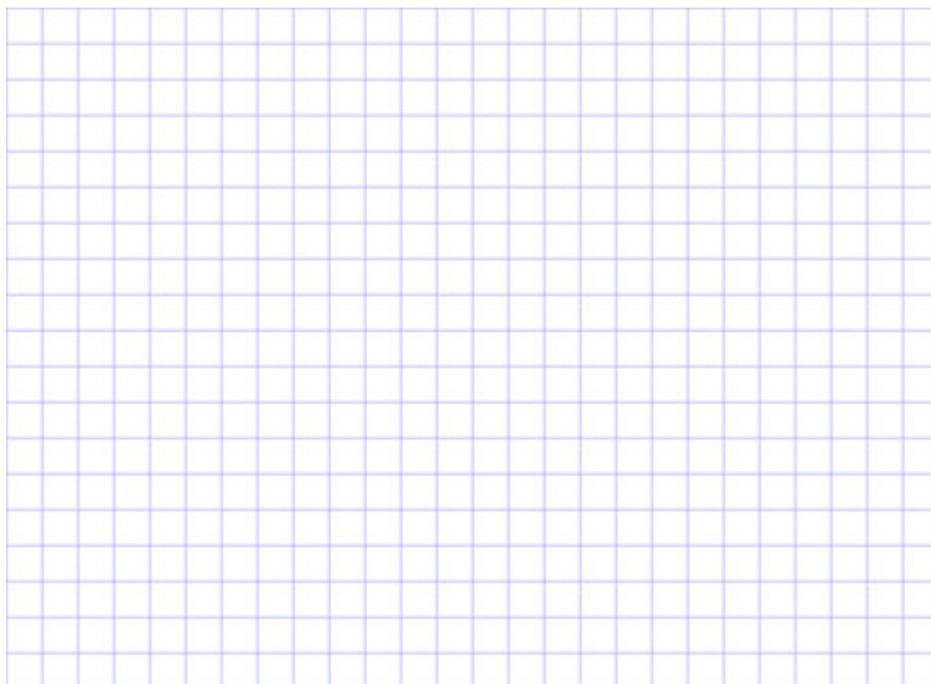
4. [**5 marks**] Let a and b be constants. Show that if the curves $xy = a$ and $x^2 - y^2 = b$ intersect each other, they do so at right angles.

5. [**6 marks**] Consider an equilateral triangle whose sides are of length 4. Determine the dimensions of the largest rectangle which may be inscribed inside the triangle if one side of the rectangle lies on the base of the triangle.

6. [**6 marks**] At what point on the interval $(0, \frac{\pi}{2})$ does the tangent line to the curve

$$y = 3x + \sqrt{3}\cos x - \sin x$$

have the smallest slope?

7. [**4 marks**] The *Bubble Nebula* is an expanding sphere of gas and stellar ejecta in the constellation Cassiopeia.

The radius of the Bubble Nebula is $3(10^{13})$ km, and it is expanding at a rate of $7(10^6)$ km/h. Determine how quickly the volume of the nebula is increasing. (Hint: the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.)

8. [**4 marks**] In mechanics, a force exerted on a steel rod is called *stress*, and denoted σ. The deformation of the rod due to this stress is called *strain*, and denoted ϵ. Stress and strain are related by the Ramberg-Osgood equation

$$\epsilon = a\sigma + b\sigma^n$$

where a, b and n are constants.

Suppose the stress applied to a steel rod increases at a constant rate of R. Determine the rate at which the strain is increasing when the stress is equal to σ_0. (Hint: your answer will be an expression involving several constants.)

9. (a) [**4 marks**] Use an appropriate linear approximation to estimate $e^{0.1}$.

(b) [**1 mark**] Is your estimate in part (a) greater than, less than, or equal to the actual value of $e^{0.1}$? Justify your answer.

This page may be used for rough work. It will not be marked.