Name (print):

ID number:

Section (circle): 001 002 003

University of British Columbia
DECEMBER EXAM for MATH 110

Date: *December 10, 2011*
Time: *12:00 noon to 2:30 p.m.*
Number of pages: *14 (including cover page)*
Exam type: *Closed book*
Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For examiners' use only		
Question	Mark	Maximum mark
1		7
2		6
3		4
4		6
5		8
6		4
7		12
8		7
9		8
10		8
11		3 (bonus)
Total		70

1. Evaluate each of the following limits.

   (a) **[2 marks]** $\lim_{x \to -\infty} \frac{5x^3 - 2x^2 + 1}{6x^3 + 7}$

   (b) **[3 marks]** $\lim_{x \to 0} \left(\frac{1}{x(1-2x)} - \frac{1}{x} \right)$

   (c) **[2 marks]** $\lim_{x \to \infty} \left(e^{-x} - x \right)$

2. (a) [**3 marks**] Define what it means for a function f to be continuous at a point a.

(b) [**2 marks**] Find the domain of the function $g(x) = \dfrac{1}{\sin x}$.

(c) [**1 mark**] Is $g(x) = \dfrac{1}{\sin x}$ continuous on the interval (∞, ∞)? Why or why not?

3. [**4 marks**] Prove that the function

$$f(x) = x^3 - 15x + 1$$

has three roots in the interval $[-4, 4]$. Make sure to state any assumptions you are making, or theorems you are using.

4. Let $f(x) = \dfrac{1}{x+1}$.

(a) [**4 marks**] Find $f'(x)$ using the limit definition of derivative. (No marks will be given for using other methods in part (a).)

(b) [**2 marks**] Confirm your answer in part (a) by finding $f'(x)$ using differentiation rules such as the Quotient Rule or Chain Rule.

5. Differentiate each of the following functions.

(a) **[3 marks]** $f(x) = \dfrac{\cos x}{\sin^2 x}$

(b) **[2 marks]** $g(x) = e^{\sqrt{x}}$

(c) **[3 marks]** $h(x) = \ln(\ln(x^2 + 2x))$

6. [**4 marks**] Let

$$f(x) = \begin{cases} \frac{1}{2}\cos x & \text{if } x < a \\ x^2 + b & \text{if } x \geq a \end{cases}.$$

Find constants a and b such that f is differentiable everywhere. Justify your answer.

7. For this question, let $f(x) = x^2 - 6x$ and let P be the point $(4, -12)$.

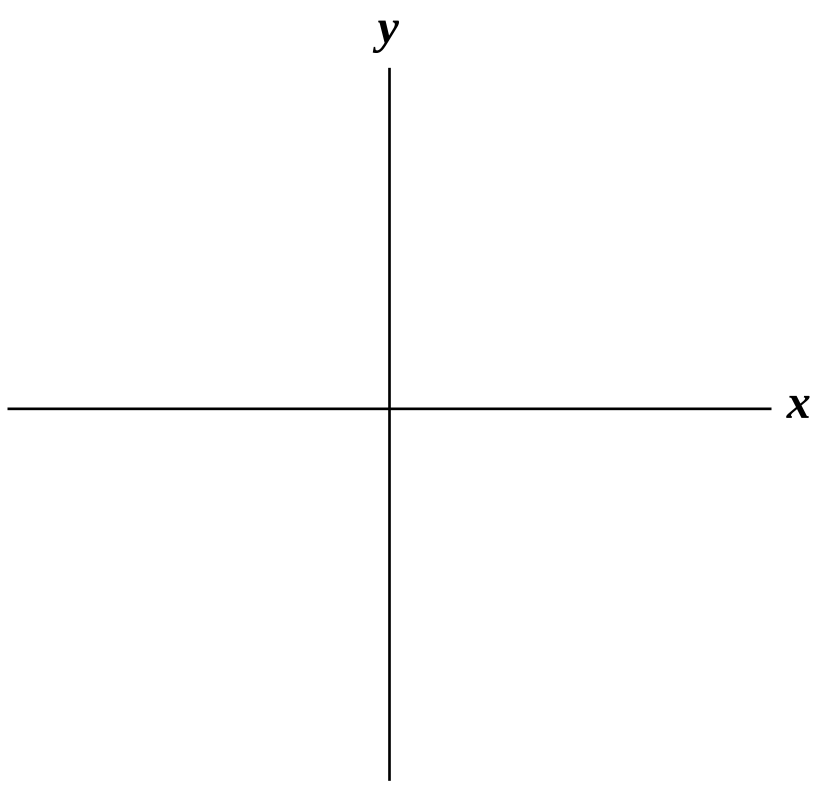
(a) [**1 mark**] Is P on the curve $y = f(x)$? Justify your answer.

(b) [**1 mark**] Find the slope of the line between P and a point $(a, a^2 - 6a)$ on the curve $y = f(x)$.

(c) [**2 marks**] Find the slope of the line tangent to the curve $y = f(x)$ at the point $(a, a^2 - 6a)$

(d) [**4 marks**] Find the slopes of all lines through P which are also tangent to the curve $y = f(x)$.

(e) [**4 marks**] Sketch, on the axes below, the curve $y = f(x)$, the point P, and the lines whose slopes you found in part (d).



8. Both parts of this question refer to the curve $y = (ax + b)^7$, where a and b are constants.

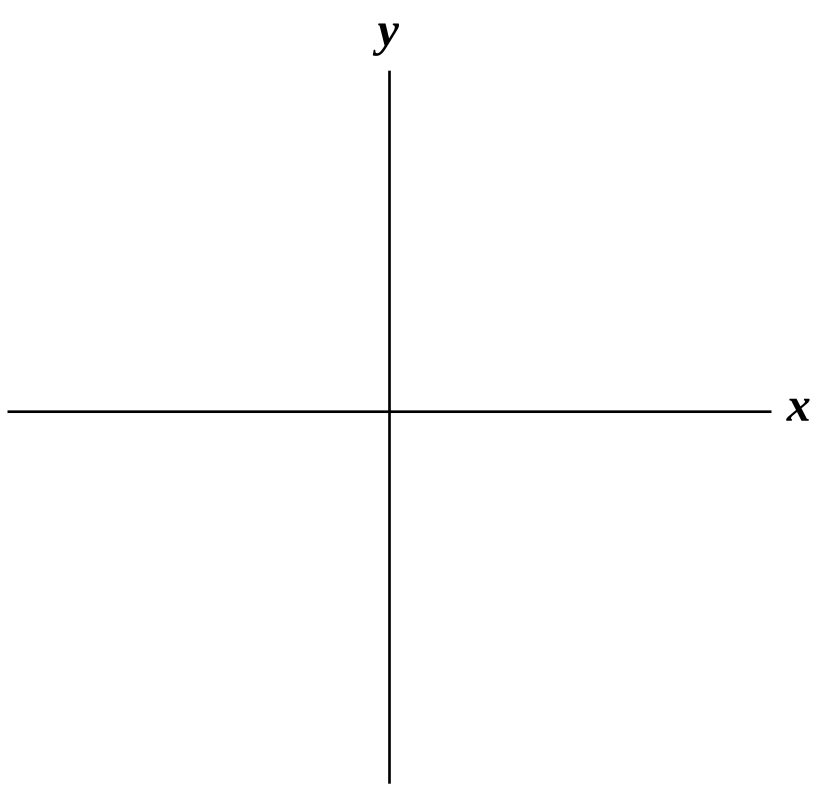
(a) [**5 marks**] Suppose that $a \neq 0$. Find the equation of the line tangent to the curve at $x = \frac{b}{a}$.

(b) [**2 marks**] Suppose that $a = 0$ in the equation for the curve given above. Find the equation of the tangent line in this case.

9. [**8 marks**] Write down an algebraic expression for a function f satisfying the following four criteria.

- $\lim_{x\to 0^-} f(x) = 2$
- $\lim_{x\to -\infty} f(x) = 0$
- $\lim_{x\to 0^+} f(x) = -1$
- $f'(x) = -2$ for $x > 0$

Then sketch the function on the axes given.



10. Let $f(x) = x + 2x^2 \sin\left(\frac{1}{x}\right)$.

(a) [**3 marks**] Find $f'(x)$.

(b) [**5 marks**] Prove that in any interval $(-d, d)$ (where $d > 0$), there are infinitely many points c such that $f'(c) = -1$. (Hint: when is $\sin x = 0$?)

11. [**3 bonus marks**] Find a piecewise algebraic expression for the n^{th} derivative of

$$f(x) = x \ln x.$$

This page may be used for rough work. It will not be marked.