

Marks

- [42] 1. **Short-Answer Questions.** Put your answer in the box provided but show your work also. Each question is worth 3 marks, but not all questions are of equal difficulty. At most one mark will be given for incorrect answers.

(a) Compute  $\frac{\partial f}{\partial x}(2, 1)$  if  $f(x, y) = e^{(1-x)y}$ .

Answer:

(b) Let  $f(x, y) = (2x + y^3)^{10}$ . Evaluate  $\frac{\partial^2 f}{\partial y \partial x}$ .

Answer:

- (c) Find all point(s)  $(x, y)$  where  $f(x, y) = x^2 + y^2 + xy + 3x - 7$  may have a relative maximum or minimum.

Answer:

- (d) Find the value of  $k$  that makes the following antidifferentiation formula true:

$$\int \frac{7}{5 - 2x} dx = k \ln |5 - 2x| + c,$$

where  $c$  is a constant.

Answer:

- (e) Suppose that the marginal revenue function for a company is  $500 - 3x^2$ . Find the additional revenue received from doubling production if currently 5 units are being produced.

Answer:

(f) Find  $\int \frac{x - 2}{(x^2 - 4x + 7)^2} dx$ .

Answer:

(g) Find  $\int 5x \sin(x + 1) dx$ .

Answer:

(h) Use the trapezoidal rule with  $n = 3$  to approximate  $\int_0^3 \frac{dx}{1 + x^3}$ .

Answer:

- (i) Find the volume of the solid of revolution generated by revolving about the  $x$ -axis the region under the graph of  $y = x^{3/2}$  from  $x = 0$  to  $x = 2$ .

Answer:

- (j) Find the area under the graph of  $y = e^{-2x}$  for  $x \geq 0$ .

Answer:

- (k) Find the consumer's surplus for the demand curve  $p = 25 - 0.03x^2$  at the sale level  $x = 10$ .

Answer:

- (l) Let  $f(x) = k\sqrt{x}$ , where  $k$  is a constant. Find the value of  $k$  such that  $f(x) = k\sqrt{x}$  is a probability density function on  $0 \leq x \leq 4$ .

Answer:

- (m) A random variable  $x$  has a probability density function  $f(x) = \frac{1}{5}$ ,  $0 \leq x \leq 5$ . Find  $b$  such that  $P_r(0 \leq x \leq b) = 0.3$ .

Answer:

- (n) Find the expected value of the random variable  $x$  whose probability density function is  $f(x) = 4x^{-5}$ ,  $x \geq 1$ .

Answer:

**Full-Solution Problems.** In questions 2–6, justify your answers and show all your work.

- [14] 2. Find the area of the shaded region bounded by  $y = 8 - x^2$ ,  $y = -2x$  and  $y = -7x$ . The shaded region is given in the following figure.

- [12] **3.** A firm makes  $x$  units of product  $A$  and  $y$  units of product  $B$  and has a production possibilities curve given by the equation  $x^2 + 49y^2 = 49000$  for  $x \geq 0$ ,  $y \geq 0$ . Suppose profits are \$3 per unit for product  $A$  and \$7 per unit for product  $B$ . Find the production schedule (i.e. the values of  $x$  and  $y$ ) that maximizes the total profit.

- [12] 4. A continuous stream of income is produced at the rate of  $10 + 2t$  thousand dollars per year at time  $t$ , and invested money earns 5% interest.
- (a) Write a definite integral that gives the present value of this stream of income over the time from  $t = 0$  to  $t = 3$  years.
- (b) Compute the present value described in part (a).

- [12] 5. A person deposits \$5000 in a bank account and decides to make additional deposits at the rate of  $B$  dollars per year, where  $B$  is a constant. Suppose that the bank compounds interest continuously at the annual rate of 8% and that the deposits are made continuously into the account.
- (a) Set up a differential equation that is satisfied by the amount  $f(t)$  in the account at time  $t$ .
  - (b) Determine  $f(t)$  (as a function of  $B$ ).
  - (c) Determine  $B$  if the initial deposit is to double in seven years.



- [8] **6.** Find a function  $f(x)$  whose graph goes through the point  $(0, 3)$  and whose slope at any point  $(x, f(x))$  is

$$\lim_{n \rightarrow \infty} \left[ \left( 1 + \left( 1 + 2\frac{x}{n} \right)^3 + \left( 1 + 2\frac{2x}{n} \right)^3 + \left( 1 + 2\frac{3x}{n} \right)^3 + \cdots + \left( 1 + 2\frac{(n-1)x}{n} \right)^3 \right) \frac{x}{n} \right].$$

Be sure that this examination has 10 pages including this cover

The University of British Columbia

Final Examination - April 27, 2009

Mathematics 105

All Sections

Closed book examination

Time: 2.5 hours

Name \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_ Instructor's Name \_\_\_\_\_

Section Number \_\_\_\_\_

**Special Instructions:**

No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form, where calculator means basic scientific calculator. Show all your work, little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

**Rules governing examinations**

**1. Each candidate must be prepared to produce, upon request, a UBCcard for identification.**

**2. Read and observe the following rules:**

Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.

No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

CAUTION - Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

(a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

(b) Speaking or communicating with other candidates.

(c) Purposely exposing written papers to the view of other candidates or imaging devices.

The plea of accident or forgetfulness shall not be received.

1		42
2		14
3		12
4		12
5		12
6		8
Total		100