Final Exam — December 18ᵗʰ 2015   Duration: 2.5 hours
*This test has **11 questions** on **14 pages**, for a total of 100 points.*

- Read all the questions carefully before starting to work.

- Q1-Q6 are short-answer questions[3pts each]; put your answer in the boxes provided.

- All other questions are long-answer; you should give complete arguments and explanations for all your calculations; answers without justifications will not be marked.

- Continue on the back of the previous page if you run out of space.

- Attempt to answer all questions for partial credit.

- This is a closed-book examination. **None of the following are allowed**: documents, cheat sheets or electronic devices of any kind (including calculators, cell phones, etc.)

- Please circle your course and section:

104	101	102	103	104	105	106	108	109
184	101	102	103	104	105	106	I don't know.		

First Name: _____   Last Name: _____

Student-No: _____   Signature: _____

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	9	6	9	9	9	6	10	10	10	12	10	100
Score:												

Short-Answer Questions. Put your answer in the box provided. Full marks will be given for a correct answer placed in the box, while part marks may be given for work shown. Unless otherwise stated, calculator ready answers are acceptable.

3 marks   1.   (a) Compute $\lim\limits_{x \to -2} \dfrac{2x^2 - 5x - 18}{x + 2}$.

Answer:

3 marks   (b) Suppose $f(2) = 3$ and $f'(2) = 2$. Let $h(x) = \dfrac{f(x)}{x^2}$. Find the equation of the line tangent to $y = h(x)$ at $x = 2$.

Answer:

3 marks   (c) Compute the derivative of $f(x) = \sin^2(e^x)$.

Answer:

3 marks   2. (a) If \$4,000 dollars is invested in a bank account at an interest rate of 6% per year, compounded continuously. How long will it take, in years, for the balance to reach \$20,000 dollars?

Answer:

3 marks   (b) Find the absolute maximum of $f(x) = \ln(x^2 + x + 1)$ over the interval $[-1, 1]$.

Answer:

3 marks 3. (a) A (spherical) balloon is being deflated at the rate of 8 cm³/s. How fast is its radius changing when the radius is 2 cm? *Note: The volume V of a sphere of radius r is $\frac{4\pi}{3}r^3$.*

Answer:

3 marks (b) Find the value of a for which the function $f(x)$ is continuous for all x.

$$f(x) = \begin{cases} x^2 + a & \text{when } x \le e, \\ 3a\ln(x) & \text{when } x > e. \end{cases}$$

Answer:

3 marks (c) Sketch a graph of a continuous function $y = f(x)$ that satisfies the following:

- $f(-5) = f(0) = 0$;
- $f'(x) < 0$ for $-5 < x < -3$ and $1 < x < 5$;
- $f'(x) > 0$ for $-3 < x < 1$;
- $f''(x) < 0$ for $-2 < x < 3$;
- $f''(x) > 0$ for $-5 < x < -2$ and $3 < x < 5$;

Clearly label and local extrema and points of inflection.
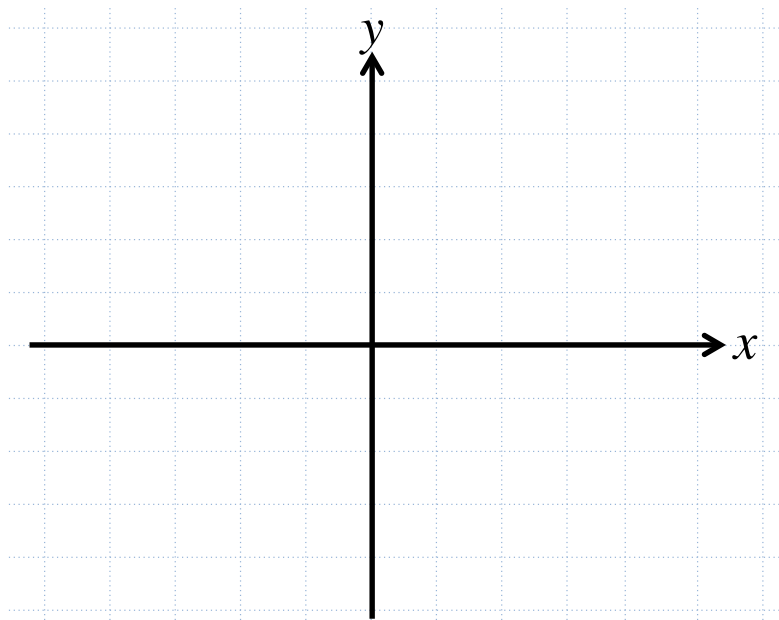
3 marks   4.   (a) Compute the 3rd degree Taylor polynomial of $f(x) = e^{-3x}$ at $a = 0$.

> Answer:

3 marks   (b) Determine an equation of the line tangent to the curve

$$x^4 - x^2 y + y^4 = 1$$

at the point $(-1, 1)$.

> Answer:

3 marks   (c) When a company manufactures x desks, the per-desk *average cost* is given by the function

$$\overline{C}(x) = \frac{50}{x} + \frac{7}{\sqrt{x}} - \frac{100}{x2^x}.$$

Use this information to determine the marginal cost function.

> Answer:

3 marks　5. (a) Use linear approximation to estimate the value of $\sqrt[3]{30}$. State your answer accurate to two decimal places.

Answer:

3 marks　　　(b) We want to estimate a solution to the equation

$$2^x = x + \pi$$

Find a positive integer n such that a solution lies in the interval $[n, n+1]$.

Answer:

3 marks　　　(c) A tailor is currently producing 80 suits per month and sells them for $100 per suit. His monthly demand curve is given by $q = 100 - 2\sqrt{p}$. Find the current price elasticity of demand and use it to decide whether price should be raised or lowered to increase his revenue.

Answer:

3 marks   6.   (a) Below is a graph of $f(x)$.
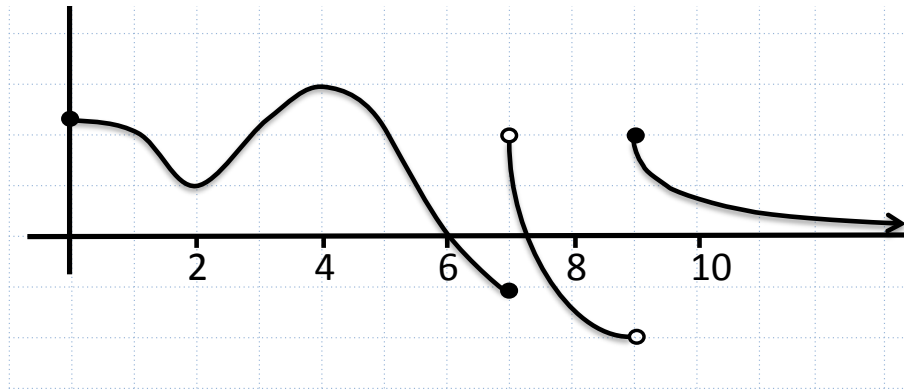
     i. What is the global maximum value?

Answer:

     ii. State the $x-$value(s) where a local minimum occurs.

Answer:

3 marks   (b) The graph of $f''(x)$ (the *second* derivative of $f(x)$) is displayed below. For what $x-$value(s) does $f(x)$ have an inflection point?
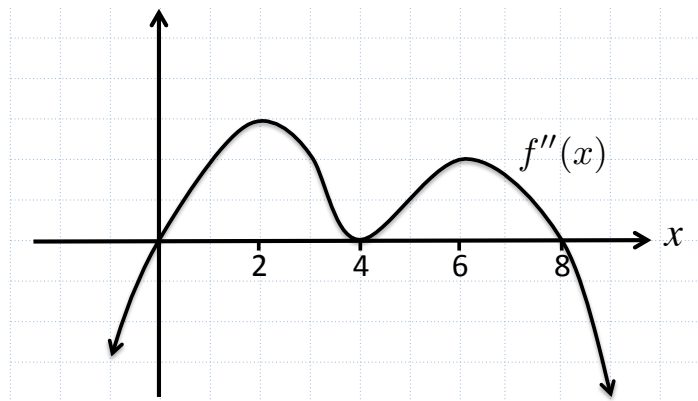
Answer:

Full-solution problems: Justify your answers and **show all your work**. Place a box around your final answer. Unless otherwise indicated, **simplification of answers are required in these questions**.

10 marks   7. Let $f(x) = \dfrac{1}{2x+1}$. Use the definition of the derivative to find $f'(2)$. No marks will be given for the use of any differentiation rules.

10 marks

8. A company is planning to produce a new electric toaster. After conducting an extensive market survey, the research department provides the following information: when the unit price is \$16 per toaster, then the weekly demand is 20 toasters; for every \$2 decrease in the unit price, the weekly demand increases by 10 toasters. The financial department estimate that the weekly fix costs will be \$140 and the variable cost of production will be \$4 per toaster. Let p be the unit price and let q be weekly demand. Note: *Please simplify your answers such that your answers for (b) and (c) are given by integers.*

(a) Find the demand function linking p and q.

Answer:

(b) What price should the company charge in order to maximize the weekly profit?

Answer:

(c) Find the break-even points for the company.

Answer:

10 marks   9. A carpenter has been asked to build an open box with a square base, where an open box
means a box without a top. The sides of the box will cost $2.50 per square meter and the
base will cost $5 per square meter. What are the dimensions of the box of maximal volume
that can be constructed for $60? Note: *In order to receive full credit, you will need to justify
your answer.*

Answer:

12 marks 10. Let $f(x) = x^2 + x - \dfrac{1}{x} - 1$. Its derivative satisfies

$$f'(x) = 2x + 1 + \frac{1}{x^2} = \frac{2x^3 + x^2 + 1}{x^2} = \frac{(x+1)(2x^2 - x + 1)}{x^2}.$$

(a) Evaluate $\lim\limits_{x \to 0^-} f(x)$ and $\lim\limits_{x \to 0^+} f(x)$.

(b) Find the intervals on which f is increasing or decreasing.

(c) Find the intervals on which f is concave up. Hint: think about what version of $f'(x)$ you would like to differentiate!

(d) Sketch the graph of f. Note: Your graph should be reasonably accurate within the grid, so you may not use the whole x-axis).
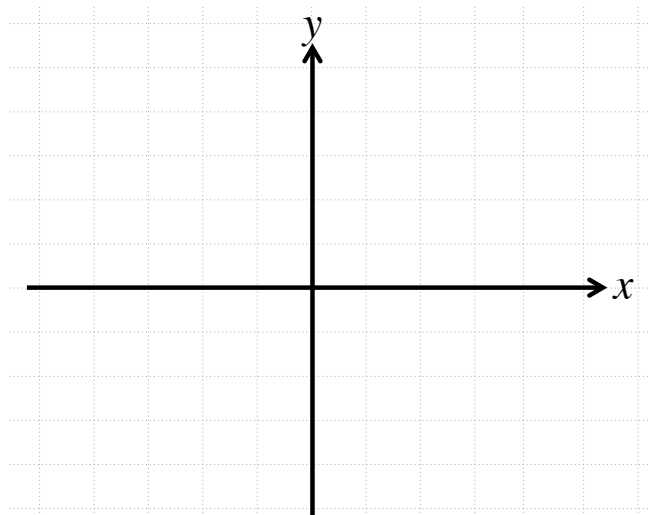
10 marks　11. A spotlight on the ground shines on a wall 15 m away. If a woman 2 m tall walks from the spotlight toward the wall at a speed of 0.7 m/s, how fast is the length of her shadow (on the building) changing when she is 8 m from the building? State your answer accurate to 2 decimal places.

Answer:

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