The University of British Columbia

Alternate December Examination 2010W

Mathematics 104/184

Closed book examination — Time: 2.5 hours

Last Name ____________ **First** ________ **Signature** ____________

Student Number ____________

Special Instructions:

No books, notes, or other memory aids are allowed. One non-programmable, non-graphing calculator is allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form, where calculator means basic scientific calculator. Show all your work, little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		42
2		12
3		12
4		12
5		12
6		10
Total		100

[42] **1**. **Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work.

(a) Evaluate $\lim_{x \to 2} \dfrac{x^2 + x - 6}{3x^2 - 5x - 2}$.

Answer:

(b) If $\lim_{x \to \infty} g(x) = 6$ and $\lim_{x \to \infty} \dfrac{g(x)}{f(x) - 1} = 3$, find $\lim_{x \to \infty} f(x)$.

Answer:

(c) How many years will it take for \$10,000 to grow to \$12,000 if it is invested at 12% annual interest compounded quarterly? You may leave your answer in calculator-ready form.

Answer:

(d) A drug that stimulates reproduction is introduced into a colony of bacteria. After t minutes, the number of bacteria is given approximately by

$$N(t) = 1000 + 30t^2 - t^3, \quad t \geq 0.$$

When is the rate of growth $N'(t)$ decreasing?

Answer:

(e) If a function $y = f(x)$ is differentiable at $x = 3$ and $f'(3) = 5$, find the limit $\lim_{x \to 3} \dfrac{x^2 - 3x}{f(3) - f(x)}$.

Answer:

(f) Find the slope of the graph of $y = \ln\big(f(2x)\big)$ at $x = 1$, where $f(2) = 3$ and $f'(2) = -5$.

Answer:

(g) You are using the Newton-Raphson Method to approximate a solution of an equation $f(x) = x$, and you make an initial guess $x_0 = 3$ to the solution. If the tangent line to $y = f(x)$ at $x = 3$ has the equation $y = 5x - 7$, what is the next approximation x_1 to the solution?

Answer:

(h) Find the equation of the tangent line to the graph of $y = x \ln x$ at $x = 1$.

Answer:

(i) If the derivative of $f(x)$ is given by $f'(x) = \dfrac{2\ln x}{x}$, find the interval or intervals on which $f(x)$ is concave down.

Answer:

(j) A racing car travels along the ellipse $x^2 + 2y^2 = 9$, where x and y are measured in kilometers. When the car is at the point $(1, 2)$, the x-coordinate of the point is changing at the rate of 3 kilometers per minute. How fast is the y-coordinate changing at that moment?

Answer:

(k) Find the global maximum and global minimum of the function $f(x) = x^{2/3}$ on the interval $[-1, 8]$.

Answer:

(l) Find the value of the constant a such that $f(x) = a\sin(2x) + x\cos x$ has a critical point at $x = \pi$.

Answer:

(m) Let $p_3(x) = c_0 + c_1x + c_2x^2 + c_3x^3$ be the third degree Taylor polynomial for the function $f(x) = e^{2x+1}$ at $x = 0$. Determine the value of c_3.

Answer:

(n) Use the linear approximation to $\ln x$ at $x = 1$ to estimate $\ln(0.8)$.

Answer:

Long Problems. In questions 2-6, show your work. No credit will be given for the answer without the correct accompanying work.

[12] **2**. (a) Find an equation of the tangent line to the curve

$$x^3 + xy^2 + y^3 = 13$$

at the point $(1, 2)$ on the curve. Please simplify.

(b) Use a suitable linear approximation (tangent line approximation) to estimate the x coordinate of the point on the curve whose y-coordinate is $\frac{31}{16}$. A calculator-ready answer is enough.

[12] **3**. A city's supply of an herb comes from many small home businesses. The supply q of the herb is a function of the street price p. The *Elasticity of Supply* is by definition equal to

$$-\frac{p}{q}\frac{dq}{dp}.$$

Suppose that a city's weekly supply q of the herb, in millions of grams, is related to the street price p, in dollars per gram, by the equation

$$p^3 + p = 2q^3 + q^2 + 10.$$

Calculate the elasticity of supply when $p = 3$. Please simplify. Note that when $p = 3$ we have $q = 2$.

[12] **4**. It costs a small firm $C(q)$ dollars to produce q kilograms of a certain chemical, where

$$C(q) = 3q^{4/3} + 50q + 10,000.$$

The *average cost* of production per kilogram is defined to be $C(q)/q$. Use calculus to find how many kilograms the firm should produce in order to minimize the average cost of production per kilogram. Please simplify. You need not justify that your calculation actually minimizes average cost.

[12] **5**. A small manufacturer wholesales leather jackets to a number of specialty stores. The monthly demand from these stores for the jackets is described by the demand equation

$$p = 400 - 50q.$$

Here p is the wholesale price, in dollars per jacket, and q is the monthly demand, in thousands of jackets. Note that the demand equation makes no sense if $q \geq 8$. The manufacturer's marginal cost function is given by the equation

$$\frac{dC}{dq} = \frac{800}{q+5}.$$

Determine the number of jackets that must be sold per month to maximize monthly profit. You do not need to justify that your answer provides the maximal profit.

[10] **6**. At a certain time, the stock of a company listed on the Vancouver Stock Exchange is selling for \$0.25 a share. The controlling shareholders spread a (quickly dispelled) rumour that the company has discovered a cure for death. This causes a surge of demand for the stock. As a result, when $t \geq 0$ the price of a share is $P(t)$ dollars, where

$$P(t) = 0.25 + 30te^{-t/5}.$$

Here t is measured in days, and $t = 0$ at the instant that the rumour starts. At what time t is the price of the stock increasing most rapidly? Please justify that the answer you calculate actually gives the greatest rate of increase.

The End