The University of British Columbia

Final Examination - December 16, 2006

Mathematics 104/184

All Sections

Closed book examination Time: 2.5 hours

Last Name ________________ **First** ____________ **Signature** ________________

Student Number ________________

Special Instructions:

No books, notes, or calculators are allowed. Unless it is otherwise specified, answers may be left in "calculator-ready" form, where calculator means basic scientific calculator. Show all your work, little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page. Where boxes are provided for answers, put your final answers in them.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		42
2		10
3		12
4		12
5		12
6		12
Total		100

[42] **1. Short Problems**. Each question is worth 3 points. Put your answer in the box provided and show your work. No credit will be given for the answer without the correct accompanying work.

(a) Find the number c that makes

$$f(x) = \begin{cases} \dfrac{x^2 - 3x - 10}{x + 2} & \text{if } x \neq -2 \\ c & \text{if } x = -2 \end{cases}$$

continuous for every x.

Answer:

(b) Evaluate $\displaystyle\lim_{t \to 0} \frac{\sqrt{t+9} - 3}{\sqrt{t}}$.

Answer:

(c) Evaluate $\displaystyle\lim_{h \to 0} \frac{(h+3)^2 - 9}{(h-5)^2 - 25}$.

Answer:

(d) If you put money in an account that pays 6% interest, compounded continuously, how long will it take for your money to triple?

Answer:

(e) Find the x and y coordinates of a point on the graph of $y = \frac{1}{4}(2x+1)^2$ where the tangent line is parallel to the line $y - 3x = 1$.

Answer:

(f) For what x does the graph of $y = e^{3x} + e^{-2x}$ have slope zero?

Answer:

(g) Let $f(x) = (\sin^{-1} x)^{-2}$, where $\sin^{-1} x$ is the inverse sine function. Find $f'\left(\frac{1}{\sqrt{2}}\right)$. Leave your answer in "calculator-ready" form.

Answer:

(h) If $f(x)$ is a function satisfying $f(0) = 1$ and $f'(0) = 4$, find the equation of the tangent line to the graph of $g(x) = \sqrt{1 + 3f(x)}$ at $x = 0$.

Answer:

(i) Let $y = y(x)$ be the function defined implicitly by the equation $y + \ln(y+3) = x^2$. Find $\dfrac{dy}{dx}$ in terms of x and y.

Answer:

(j) Determine where the function

$$f(x) = \frac{1 + \ln(x+1)}{x+1}, \quad x > -1$$

is increasing.

Answer:

(k) Determine where the function $f(x) = \dfrac{x}{x+2}$, $x \neq -2$, is concave down.

Answer:

(l) Find the global minimum of the function $f(x) = \dfrac{x}{x^2+4}$ over the interval $[-1, 5]$.

Answer:

(m) Determine the sum $-4\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right)^2 - 4\left(\frac{1}{5}\right)^3 + 4\left(\frac{1}{5}\right)^4 - \ldots$.

Answer:

(n) Let $c_0 + c_1 x + c_2 x^2 + \ldots$ be the Taylor Series of the function $f(x) = (x+1)e^{-x}$ at $a = 0$. Determine the value of c_2.

Answer:

Long Problems. In questions 2 - 6, show your work. No credit will be given for the answer without the correct accompanying work.

[10] **2**. Use the definition of the derivative as a limit to find the derivative of $f(x) = \dfrac{x}{1-3x}$, $x \neq \dfrac{1}{3}$. No marks will be given for the use of differentiation rules.

[12] **3**. Suppose that when a busy restaurant charges \$7 for its tomato appetizer, an average of 60 people order the dish each night. When it drops the price of the appetizer to \$5, the number ordering it rises to 66. Assume that the demand q is a linear function of the price p. If each appetizer costs the restaurant \$3 to make, use calculus to find the price it should charge to maximize its profit from the appetizer. You do not need to justify that your answer provides the maximum profit.

[12] **4**. Let $y = f(x) = \dfrac{2x}{x^2+1}$.

(a) Find the interval or intervals on which $f(x)$ is increasing. [3pts]

(b) Find the interval or intervals on which $f(x)$ is concave up. [3pts]

(c) Sketch the graph of $y = f(x)$, and indicate the x-coordinates of any inflection points, and the values of x where any global maxima or global minima occur. (Hint: $\sqrt{3} \approx 1.7$) [6pts]

[12] **5**. A carpenter has been asked to build an open box with a square base, where an open box means a box without a top. The sides of the box will cost \$2 per square meter, and the base will cost \$5 per square meter. What are the dimensions of the box of maximal volume that can be constructed for \$60 ? You do not need to justify that your answer provides the maximal volume.

[12] **6**. The price p (in dollars) and demand q for a product are related by

$$p^2 + 2q^2 = 1100.$$

If the price is increasing at a rate of \$2 per month when the price is \$30, find the rate of change of the revenue in dollars per month.

The End