The University of British Columbia

Final Examination - April 28, 2010

Mathematics 103

Instructors: Dr. Israel, Dr Hauert, Dr. Coombs, Dr. Das, Dr. Keshet **(Circle one)**

Closed book examination Time: 2.5 hours

Name (Last, First): ______________________ **Signature** ______________________

Student #: ______________ **Section:** 201/ 202/ 203/ 204/ 205/ 206 **(Circle one)**

Special Instructions: - Be sure that this examination has 10 pages. Write your *full* name (as on your Student ID) on top of each page. You may use backs of pages for scrap work, (or extra space, if needed).

- No calculators, electronic devices, books, or notes are permitted.
- Unless otherwise indicated, show all your work. Answers not supported by calculations or reasoning may not receive credit. Messy work will not be graded.
- At the end of the examination period: 1. You will be instructed to put away all writing implements (Continuing to write past this signal is considered cheating). 2. Remain in your seats until exams have been collected. 3. You will be instructed when you are free to leave.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

  (a)having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;

  (b) speaking or communicating with other candidates; and

  (c) purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		16
2		12
3		12
4		12
5		12
6		12
7		12
8		12
Total		100

Problem 1: Multiple Choice Questions: Circle ONE correct answer (a, b, c, d, or e). There is no partial credit in this question.

1: To which of the following integrals does the Fundamental Theorem of Calculus apply?
(a) $\displaystyle\int_{-1}^{1} \frac{\cos(x)}{\ln(x)}\,dx$
(b) $\displaystyle\int_{-1}^{1} \frac{\cos(x)}{x}\,dx$
(c) $\displaystyle\int_{-1}^{1} \frac{\cos(x)}{\sqrt{x}+1}\,dx$
(d) $\displaystyle\int_{-1}^{1} \frac{\cos(x)}{x^2+1}\,dx$
(e) None of the above.

2: To find the integral $\displaystyle\int x^2 e^{2x}\,dx$ using integration by parts, the best choice for u and dv in the first step would be
(a) $u = x$, $dv = xe^{2x}\,dx$
(b) $u = x^2$, $dv = e^{2x}\,dx$
(c) $u = xe^{2x}$, $dv = x\,dx$
(d) $u = e^{2x}$, $dv = x^2\,dx$
(e) $u = x$, $dv = e^{2x}\,dx$

3: The expression $S_3 = 1 + x^3 + \frac{x^6}{2}$ is the first three terms of a Taylor series for:
(a) $\sin(x^3)$
(b) $\cos(x^{\frac{3}{2}})$
(c) $\cos(x)$
(d) $\cos(x^3)$
(e) e^{x^3}

4: Which of the following series converges?

(a) $S = \displaystyle\sum_{k=1}^{\infty} k^{-2}$ (b) $S = \displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$ (c) $S = \displaystyle\sum_{k=1}^{\infty} k$ (d) $S = \displaystyle\sum_{k=0}^{\infty} (1.01)^k$ (e) $S = \displaystyle\sum_{k=0}^{\infty} k^2$

Problem 1, Cont'd from page 2:

5: Which of the following improper integrals converges?
(a) $I = \int_1^\infty e^{3x}\,dx$
(b) $I = 10^{-5} \cdot \int_1^\infty x^{-1}\,dx$
(c) $I = \int_1^\infty x^{-1/2}\,dx$
(d) $I = \int_1^\infty x^{-2}\,dx$
(e) $I = \int_1^\infty \frac{x^2}{1000}\,dx$

6: A continuous random variable X has a probability density of the form

$$f(x) = Ax + Bx^2, \quad \text{for} \quad -1 \le x \le 1$$

where A and B are constants. The values of A and B should be
(a) $A = 1, B = 1$
(b) $A = 1, B = 3/2$
(c) $A = 0, B = 3/2$
(d) $A = 0, B = 2/3$
(e) $A = 2, B = 3$

7: A random variable X can take on one of the values $\{1, 3, 5\}$ with (discrete) probabilities $p(1) = 1/2, p(3) = 1/4, p(5) = 1/4$. The mean value of X is then

(a) $\frac{5}{2}$ (b) 9 (c) 3 (d) $\frac{2}{5}$ (e) 1

8: Consider the differential equation $\frac{dy}{dt} = 1 - 4y^2$ and initial condition $y(0) = 0$. After a long time (when y no longer changes), the value of y is

(a) $y = 0$ (b) $y = 1$ (c) $y = 1/4$ (d) $y = 4$ (e) $y = 1/2$

Problem 2:

(a) Calculate the area of the bounded region enclosed by the curves $f(x) = x^3 - 4x$ and $g(x) = -x^2 + 2x$ for $x \geq 0$.

Problem 3: A toxic chemical has been spilled somewhere on a 100 km stretch of highway. The probability that the spill occurred within x km of the factory is shown in the top panel of Figure 1.

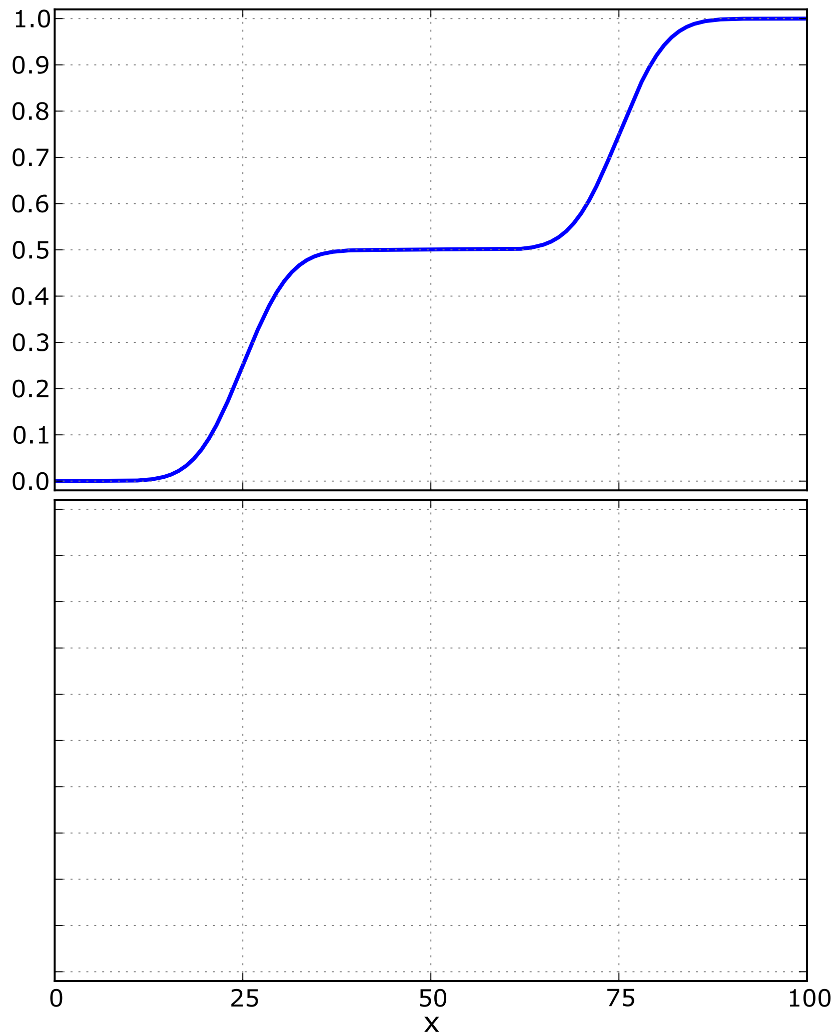

Figure 1: Plot and grid for Problem 3

(a) Plot the probability density $p(x)$ for the location of the spill in the grid provided on the bottom panel. (You do not need to provide a scale on the y axis.)

(b)What is/are the most probable location(s) of the spill?

Problem 4: Compute the following integrals. (Leave your answers in terms of e or $\ln()$.)

(a) $I = \displaystyle\int_4^9 e^{\sqrt{x}}\, dx$

(b) $I = \displaystyle\int_e^{e^2} \frac{1}{x\ln(x)}\, dx$

(c) $I = \displaystyle\int_0^1 \frac{x}{(2-x)(1+2x)}\, dx$

Problem 5: In this problem, you are asked to find a value for the integral

$$I = \int_0^1 x\cos(x)\,dx$$

in two different ways.

(a) Use one of the integration techniques to compute the above integral directly. (Leave your answer in terms of sines and cosines of some number.)

(b) Use the Taylor series for the function $\cos(x)$ to write down a Taylor Series approximation for $x\cos(x)$ and find an approximation to the integral I using the *first 3 terms* of that series. (Leave your answer in terms of the three fractions, i.e. you need not compute a simplified fraction nor a decimal approximation. See footnote[1] for some factorial values.)

[1] $2! = 2$, $3! = 6$, $4! = 24$, $5! = 120$, $6! = 720$, $7! = 5040$

Problem 6: The height of fluid, $h(t)$ in a cylindrical container is controlled by a pump so that it satisfies the differential equation

$$\frac{dh}{dt} = -kh^{1/3}, \quad h(0) = h_0$$

where $k > 0, h_0 > 0$ are constants, and h_0 is the initial height of the fluid.

(a) Solve this differential equation to determine the height $h(t)$ at any later time t.

(b) At what time will the container be empty?

Problem 7: The probability that a newly divided cell will divide again **before** t hours (where $t \geq 0$) is given by

$$F(t) = 1 - e^{-t/12}$$

(a) If you start observing a cell immediately after a division, how long, on average, would you have to wait to see the next division?

(b) What is the probability that the cell will divide between 3 and 6 hours? (Leave your answer in terms of e.)

(c) What is the median division time?

Problem 8: The graph of the function

$$y = f(x) = x(1 - x), \quad 0 \leq x \leq 0.5$$

is rotated about **the y axis** to form a trumpet shape whose base is at (0,0). Determine the volume of fluid that can be contained inside this shape. (**Hint:** One way to solve this problem is to find the function $x = g(y)$ that describes the same curve and use it to set up the appropriate integral.)